“Mathematics education is much more complicated than you expected, even though you expected it to be more complicated than you expected.”

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Introduction

In April of 2018, Canadian mathematics educators gathered together at the Fields Institute in Toronto, Ontario to discuss issues surrounding the teaching and learning of first-year post-secondary mathematics courses across Canada. The conference included discussions, presentations, and working groups. These sessions focused on consolidating information from the First Year Math in Canada course repository and on sharing innovative teaching and course development practices.
Working Group Reports

During the conference, participants broke into 10 working groups, looking for information and common themes that appear in the First Year Math in Canada course database, with an aim to paint a portrait of how we teach first-year math courses, today, in 2018.

The following summarizes key ideas from each working group:

**Group 0: Teaching Linear Algebra**

- The topics covered in Linear Algebra vary from course to course and institution to institution, as do class sizes, specialized version of the course (i.e., for business students), and connections to future courses.
- Most versions of Linear Algebra are identified as ‘service’ sections (not for math majors only), and a big subset these courses are not ‘terminal’ in nature.

**Group 1: Global View of Courses**

- High school prerequisites for first-year math courses vary greatly across the country and are not standardized, making it difficult to interpret pre-requisite information from different provinces and regions.
- There is a large amount and proportion of service teaching happening at the first year level and many universities offer service courses that are ‘math appreciation’ style courses. These courses tend to be terminal in nature, while Calculus and Reasoning and Proof style courses tend to be ‘gatekeepers’ to higher level math courses.
- Calculus is still very much the dominant course in first-year math across Canada, both in terms of courses and student numbers. However, specialized Calculus courses are changing (diluting) the type of Calculus that dominates (pure versus applied).

**Group 2: Teaching Strategies**

- Lecturing remains the primary form of instruction in first-year math across Canada, but most institutions offer lectures plus tutorials, workshops, and other resources for students.
- Certain universities offer alternatives to lecturing, such as online courses, flipped classes, and inquiry-based practices. Technology in teaching is growing via practices such as student response systems in class and online homework.

**Group 3: Teaching Materials**

- Instructional materials include traditional textbooks, open-source texts, custom lecture notes, and learning management systems. Of the textbooks, Stewart is the most frequently used.
- About half of the courses listed in the database include some online component: online homework system through publishers such as WebWorks, WebAssign, MyMathLabs, etc. or an in-house custom version.
- Most frequently, heavily weighted parts of the courses are mostly paper based - term tests and final exam.

**Group 4: Curriculum Across Universities**

- Calculus and Linear Algebra are the ‘bread and butter’ of first-year math across Canada.
- There is variance in terms of content covered within these courses.
- Interdisciplinary courses seem to appear in the database.
Group 5: Calculus Content Across Universities

- Though Calculus ‘specialization’ varies from institution to institution (with larger institutions offering more specialized versions of Calculus), there is a standard set of Calculus topics that is covered in each course.
- Advanced calculus is offered at some institutions, where more of a theoretical or real analysis perspective is accompanied in the standard calculus offered to math and statistics specific students.
- Very few calculus classes are single and terminal courses (only for forestry, etc.). The majority of Calculus in the first year is offered in courses with levels (Calculus 1 and 2) that span over two or more terms.
- Approximately half of the institutions in the database offer Pre-Calculus. Some even offer a Pre-Pre-Calculus course.

Group 6: Computer Science Courses

- The integration or separation of computer science and mathematics courses varies across Canada.
- It seems that most computer science courses focus on programming. The languages used include Python, Java, C, C++, and Racket. Math courses use MATLAB, Maple, R, Excel, SAS, SPSS, and Python.
- Mathematics students in most institutions across Canada are required to take some computer science courses as a part of their program.

Group 7: ‘Outlier’ Courses

- Such outlier courses can be used as breadth requirements that help to build confidence and that act as an outreach or a bridge to help students develop an appreciation for mathematics.
- Outlier courses can be highly instructor specific and dependent. They can be difficult to create and staff.

Group 8: Patterns in Assessment Methods

- Most assessments are still traditional (paper and pencil examinations), with final examinations counting for a high proportion of the grade weight in most courses.
- Most first-year courses have other assessments as well that range from tests to quizzes, worksheets, online assignments, workshops, and projects.
- Institutions are sporadically trying out different testing techniques, such as group exams, to better align the assessment methods with classroom activities.

Group 9: State of Math and the Present Work Force

- Many programs/fields are served by our first-year math courses (business, life sciences, chemistry, biology, computer science, engineering…).
- We struggle to help our students to see the value of mathematics in the real world and to articulate to them the types of marketable skills that they are developing in our courses. A focus on employment beyond university is not apparent from the database.
Important Ideas Identified

Following the working groups, key ideas/statements were identified and posted throughout the room for participants to vote on using a DOTmocracy approach. These were statements and future directions that the groups felt were important for discussion, either in agreement or disagreement.

Each participant was given five dots to place next to the ideas that they deemed to be most important in the near future.

The themes/statements that received a larger proportion of votes are shown in the following chart:

Teaching materials, active learning approaches, interdisciplinary courses, assessment, and student maturity were the themes identified as most important for further analysis and exploration. Further details about voting are given in the table on the next page.
What is Not There

As most ideas were extracted from or were related to the data from the Repository, the above results capture some aspects of teaching mathematics in 2018. A comprehensive list of ideas, teaching strategies, curriculum modifications, etc. would be much longer and include: vertical integration of mathematics curriculum, incorporating computational thinking and programming into math courses, modernizing statistics to include data science, teaching geometry and spatial reasoning, teaching numeracy and relation between numeracy and mathematics/statistics, and so on. As well, we did not place nearly enough attention to the situation and problems that are specific to our smaller universities.