

# First-Year Mathematics Repository Workshop (19w2256)

## Report

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Workshop Notes by

Darja Barr and Amanda Malloch

Appendices by

Darja Barr, Amanda Malloch, Michelle Davidson, Patrick Maidorn, Barbara Forest,  
Fok-Sheun Leung, and Kseniya Garaschuk

## Introduction

The Banff International Research Station hosted the "First Year Mathematics Repository Workshop" workshop in Banff, Alberta from 8 to 10 February 2019. In attendance were 24 participants, representing 12 universities (the majority western universities): University of Alberta, University of Calgary, University of Victoria, University of Fraser Valley, Simon Fraser University, University of Regina, Mount Royal University, University of Manitoba, Queen's University, University of Toronto Mississauga, University of Waterloo, and McMaster University.

The Banff workshop was an important event within an ongoing National dialogue on the present and future of teaching first-year mathematics at Canadian universities. The dialogue, initiated in the article in CMS Notes (Oct/Nov 2017) written by V. Jungic and M. Lovric, takes different forms: from informal meetings of colleagues from the same department to discussions and exchanges of ideas over social media; and from conducting surveys and writing academic papers to regional and national workshops and conferences. The workshop was built on, and informed by, the outcomes of the CMS Winter Meeting Education session in Waterloo, Ontario (9 December 2017) and the First Year University Mathematics Across Canada: Facts, Community and Vision conference at the Fields Institute in Toronto, Ontario (27-29 April 2018). The follow-up events are planned at the University of Alberta (3-5 May 2019) and the University of Toronto Mississauga (22-24 May 2020).

The themes that were addressed in the plenary sessions, working groups and other activities related to the so-called "service courses," perhaps best described as (often massive) courses offered by mathematics departments that serve students other than mathematics or statistics majors. 'How do different institutions and different instructors find the balance between introducing a relatively complex math content and meeting the needs of the specific program that the course "services"?' was the guiding question. More specifically, the workshop participants addressed the

following questions:

1. What makes a math/stats service course program specific, and what are the challenges?
2. To what extent (if at all) are the service courses outcomes of the collaborations between the department of mathematics and/or statistics and another academic unit?
3. How do we address the diversity of the student populations within service courses?
4. What are the available learning resources for service courses?

The Banff workshop also served as a crucial step in further promotion and development of The First Year Mathematics and Statistics Courses Repository, a resource supporting an ongoing national dialogue about teaching first-year mathematics and statistics at Canadian universities. This shareable dynamic online database contains extensive data, collected from mathematics instructors across the country, including course content, resource and technology used, learning outcomes, modes of delivery, connections with other courses, as well as informal descriptions of various practices in teaching these courses.

### **Executive Summary:**

- Math and Stats departments must pay lot more attention to their service courses.
- Service courses provide unique opportunities to teach mathematics that is interesting, exciting, and stimulating, and that addresses authentic life situations. This is where we are forced to re-think the mathematics content, to benefit not only service courses, but all math courses.
- Service courses are mostly taught by younger instructors; often they are on limited-term contracts or hold more permanent, but non-tenure track positions.
- Instructors teaching service courses bring huge amounts of enthusiasm and energy into their courses. They are willing to experiment with a variety of pedagogical approaches and technology (plenty of evidence to this presented at the workshop).
- A successful design of a service course requires continuous communication with faculty in all departments whose students will be taking the course. Course design is an intense, time-consuming process, and those involved in the design should be given time necessary to develop the course.
- Much-needed innovation in math and stats instruction happens in service courses! This is a major reason why math and stats departments across the country should pay more attention to these courses, as well place more resources (human and money) into them.
- For the reasons mentioned above and given the stigma that comes with the attribute “service,” it might be a good idea to find a more suitable name; for

instance, to rename “service courses” into “mathematics and statistics courses,” and refer to their complement as “courses for mathematics and statistics majors.”

### **Workshop activities and discussion:**

The Banff workshop started on Friday, 8 February, in the afternoon. Although no formal activities were planned, the time was used for introductions and much needed socializing and informal conversations. As is well known (and thus was built into the workshop), many important discussions, exchanges of views and ideas, and building of foundations for future collaborations take place outside of the formal workshop/conference activities. The lounge in Corbet Hall, the fireplace area in particular, proved to be an ideal facilitator for this. Moreover, this informality encouraged openness and the depths of critique that were definitely embraced by the participants, who are weary of daily feeds of statements, documents, and views (mis)guided by university branding proclamations, public relations departments, and political correctness.

Extensive notes on the workshop proceedings were compiled by Darja Barr and Amanda Malloch (Big Thank You!!) and are posted as separate files.

The reports of the four working groups appear in appendices. They were compiled by Darja Barr and Amanda Malloch (Service courses for physical sciences and engineering students; Appendix A); Michelle Davidson and Patrick Maidorn (Service courses for arts, humanities, social sciences, and business students; Appendix B); Barbara Forrest (Service courses for mathematics education students; Appendix C); and Fok-Sheun Leung (Service math and stats courses for life sciences students; Appendix D).

Instead of commenting on the workshop activities one-by-one (that is done in the appendices), we present some major points that were discussed.

Although service courses “service” thousands of students, they are not given the attention, nor resources that they deserve and need, by their home departments or faculties. (Some participants argued that there would be no math and stats departments without the money that the service courses earn.)

Junior faculty and sessional instructors, who often teach such courses, are given large teaching loads (5 or up to 7 courses per academic year) with demands on their time that are so severe that they leave very little (or no) time for anything else. However, in spite of this, and with their work penetrating deep into their private and family lives, they innovate, experiment, and put energy into improving the courses they teach. Given extra time, we all could do a lot more – for instance, to have an informed design of a service course, we must communicate with the members of relevant departments across campus. This does not involve a couple of meetings, but rather a continuous effort.

The applications of mathematics and statistics taught in a service course need to be authentic to the students in order for the service course to have value. Needless to say, we are not assuming that a service course for life scientists will educate future researchers in mathematical biology, but can nevertheless bring the applications we study closer to reality. In presenting mathematical and statistical models in our courses, we need to be clear about the assumptions that were made, and about the limitations, both with regards to the situation modeled and the mathematical/statistical tools used. See, for instance, suggested activities created by Kseniya Garaschuk in Appendix E.

Although often, when we say “mathematics”, we mean “mathematics and statistics,” this implicit assumption hurts statistics and misguides our students’ perceptions of its importance (and increasing relevance in today’s society). This situation is further exacerbated by the content of some service courses we teach: for instance, a business or a social sciences calculus course could profit by replacing at least half of its content by important and relevant statistics concepts.

Service courses “attack” the thinking of what a math or stats course should contain. A mathematician would never put complex integration, boundary value problems and several topics in linear algebra into one course – yet, it is what engineering departments routinely ask math and stats departments to do. But perhaps this is a friendly advice instead of an attack?

A service course does not have to be a course in a traditional sense. If we think of it in this way, new opportunities arise – why cannot we have a two-week module (“just-in-time”) that can help students in their mechanics course with integration, as they need it before it is covered in the calculus that they are taking? There are initiatives along these lines: at the University of Toronto Mississauga, students have optional modules, created in collaboration and taught by a mathematician, on double and triple integrals and vector calculus for several physics courses and on exponential functions and population dynamics for biology students.

We have heard a number of participants acknowledging the value of the data in our *First Year Mathematics and Statistics Courses Repository*. However, to make it a truly beneficial resource, we need to continue working on it. We all agreed that we have to maintain and update the information in the *Repository*, as well as make the search function more efficient, for instance by including key words.

### **Conclusion and future related activities:**

With this successful start in mind, our next focus in terms of discussing service courses will focus on action, collaboration and creativity. Among other activities, through our future workshops and conferences we plan to:

1. Establish communication with relevant departments and include their members in our discussions about service courses (we will invite instructors and researches from biology, chemistry, business, psychology, etc.).

2. Enhance collaborations between service courses instructors that go beyond a single university, For instance, a group of universities could collaboratively create and teach a course in math and stats for life sciences, or business, or social sciences students.
3. Look into enhancing collaborations with teaching and learning units/ departments/ centres at universities.
4. Take a deep look into available resources, and use our *Repository* as means of making them easily accessible.
5. Work on creating courses that are service, but in a broader sense – such as a “big ideas in math and stats” course.
6. Conceptualize and teach numeracy as a part of a service course.
7. Lobby math and stats department chairs and associate chairs to place more resources into service courses.

We have received numerous, overwhelmingly positive comments and praise for the workshop. We quote two of these:

“[...] it is really great to see all of you here. People are really passionate about teaching, and people who actually care about improving the university educational experience for students and making math courses more engaging and thoughtful.”

“Of the many education conferences that I have attended in the past few years, the First Year Math Repository conference just seems to have a special sense of camaraderie, mutual trust and respect among participants which permits everyone to discuss any point of view.”

It is important to emphasize that the single most important reason why the Banff workshop was deemed a success lies in its participants. Lots of work is ahead of us, but with so many highly enthusiastic and energetic colleagues, we are confident that we will be able to make improvements in the ways math and stats courses are taught in our universities.

# WG Service courses for physical sciences and engineering students

## Report

From Notes by Amanda Malloch:

### • Calc for Engineering & Sciences.

- coordination of exams
  - ↳ maybe one question different for instructors
- Applications?
  - ↳ should there be more
- Guest speakers on applications.
- Should we teach other things in first year?

- ↳ Mathematical Thinking
- ↳ Shared course across Canada
- ↳ focus on a few main theorems. (in different areas). & proofs.

Big Ideas Course.  
Book: Beyond Numeracy

- Just-in-time teaching
  - ↳ is this good?
  - ↳ necessary for some disciplines.
- Meetings with multiple instructors outside of class time to make connections across disciplines.
- Would be a big loss if we lost some service courses if the home department decided to teach them instead
- Tutorials?
  - ↳ Present a poster (Calgary)

\*we're currently doing something in the middle.

From Notes by Darja Barr:

Working Group Reports:

- *Service Courses for Physical Sciences and Engineering*: We all do this and we all do it differently in terms of things like class size, assessments, exam formats, tutorials, etc.

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Applications are desirable, but only if they are authentic, like project based applications. The idea of guest speakers was raised as a potential source to tap. Most courses try to do a combination of 'Just-In-Time' teaching of necessary skills and mathematical reasoning. There is a danger of other departments taking over these service courses, which was agreed to not be a very good thing.

First Year Math Repository Workshop  
February 9-10, 2019

Working Group - Combined Group: Arts, Humanities, Social Science and Business Students

Institutions Represented:

- \* Mount Royal University
- \* Simon Fraser University
- \* University of Manitoba
- \* University of Regina
- \* University of Toronto Mississauga
- \* University of Waterloo

Courses for Business:

- \* Calculus
- \* Statistics
- \* Financial Math
- \* Matrices for Management (defunct)
- \* Separate from course offerings for Actuarial Science students

For Arts, Social Sciences:

- \* Statistics
- \* Logic/Problem Solving
- \* Math Appreciation/Links (Math and Art, ...)

Notes on courses:

- \* Calculus appears to be still used as a “filter” class
- \* Material in course not necessarily relevant to later business/economics classes

Question: Who should make the point of deciding on what’s in the classes - same material for 25 years. Collaboration with other department/faculties are often minimal or non-existent. In some cases, requests from other faculties seem counter-intuitive to our perception of needs for students - e.g. request not to include any statistics for business students.

How are courses differentiated from those for other streams?

- \* Primarily in use of examples (e.g. optimize a profit function instead of optimize a physical or geometric quantity)
- \* Some variation in material or order in which material is presented (e.g. early introduction of Lagrange multipliers)
- \* Early introduction of exponential/logarithmic functions, often no introduction of trigonometry
- \* (now defunct) attempt at early introduction of sequences instead of continuous functions

How to engage students

- \* relevant authentic questions - involve other departments, guest lectures, “real” problems
- \* applied project/example - “make your own company”



- \* don't put the math first, put the context/question first
- \* interpret answers, put back into context
- \* role of the repository for relevant problems and other material - no need to re-invent the wheel at each institution

Resources available to students:

- \* Online Help
  - \* Drop in help centres (sometimes course specific)
  - \* Discussion forums with TA monitors
- (Note: none truly differentiated from resources available for other streams)

Evaluation methods and related resources (beyond exams):

- \* Worksheets - not graded (just complete/not complete)
- \* Assignments - not graded/graded, current experiment: half the class get assignment for grades, the other half not.
- \* Online assignments true/false/multiple choice. Small but very frequent.
- \* Big List of Problems - 50% of questions taking from that - alleviate the "unexpected"
- \* Quizzes, e.g. "Best 4 of 5" or "Best 3 of 5 - but one must be from last two"
- \* Crowdmark

**A Discussion About  
First Year Math Service Courses  
for  
Education Students**

February 10, 2019

Should we set minimum standards that elementary students should meet in mathematics?

If so, what percentage of students should meet these standards?

In Ontario,

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There is research that suggests **the lack of STEM aspirations can be linked to poor mathematics confidence**. Not science, not technology, not so much engineering, its math.

**Reference:** *Fuzzy Numbers: How Math Instruction Varies widely for teachers to be across Canada*: Globe and Mail: August 14, 2018

For the past 4 years, an Ontario teachers' college has administered a Grade 6-7 level math test to Education Students.

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The following slides provides sample questions from these tests.

**Reference:** *Fuzzy Numbers: How Math Instruction Varies widely for teachers to be across Canada:* Globe and Mail: August 14, 2018

QUESTION #1

**Divide a 4-digit number by a 2-digit number  
to the hundredth decimal**

$$1246 \div 45$$

a.24.92

b.27.69

c.33.28



QUESTION #2

**How much chocolate is left?**  
**Represent in a fraction**



a.  $\frac{7}{16}$

b.  $1\frac{1}{4}$

c.  $1\frac{3}{8}$

QUESTION #3

**Convert a percent number  
into a fraction**

**167%**

- a.  $1\frac{67}{100}$
- b.  $16\frac{7}{10}$
- c.  $160\frac{7}{100}$

QUESTION #8

**Calculate the percentage  
of a whole number**

**55% of 110**

- a.55
- b.56.5
- c.60.5

QUESTION #9

Use order of operations  
to solve

$$13 - 6 + 8$$

a.-1

b.3

c.15

QUESTION #10

**Solve problem using numeracy skills  
in percent, fractions, and/or decimals**

**Kiana has read 120 pages of her book – 120  
pages equals to 40% of the entire book. How  
many pages does she have left to read?**

- a. 100 pages
- b. 120 pages
- c. 180 pages

What math content knowledge should elementary teachers know?

What percentage of teaching candidates should meet this standard?

Approximately **one-third** of the teaching candidates scored at or below 70 percent, the provincial standard for Grades 6-7 mathematics.

**Reference:** *Fuzzy Numbers: How Math Instruction Varies widely for teachers to be across Canada*: Globe and Mail: August 14, 2018

# How Math Instruction Varies for Teachers-to-be Across Canada

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A Globe and Mail analysis has found the amount of classroom time elementary student teachers spend on math in university training programs across the country varies from

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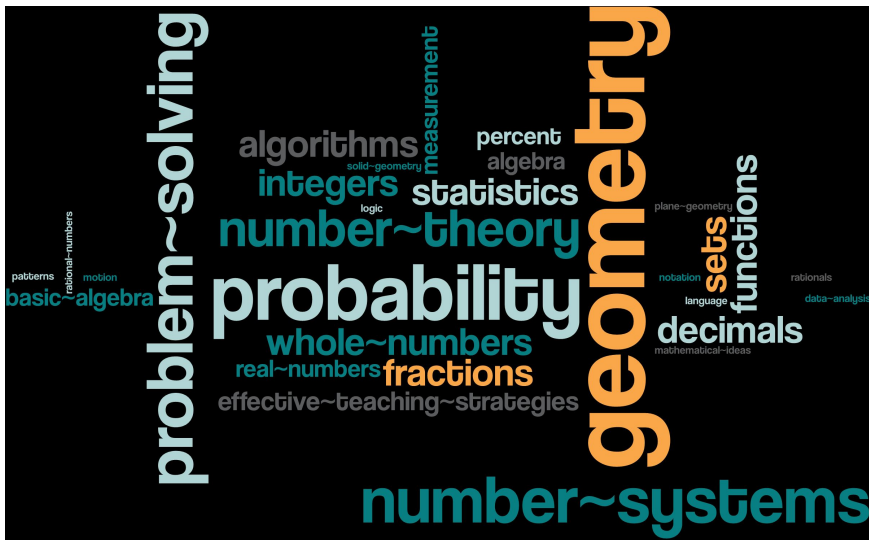
**as few as 36 hours to more than 100 hours.**

Further, the focus in **teacher education programs primarily involves learning how to teach the subject**, with less time spent re-learning math concepts.

**Reference:** *Fuzzy Numbers: How Math Instruction Varies widely for teachers to be across Canada:* Globe and Mail: August 14, 2018

# Main Topics of Study in Math Ed Courses?

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\*Reference: First Year Mathematics Repository:  
search on “math teacher” at [firstyearmath.ca](http://firstyearmath.ca)

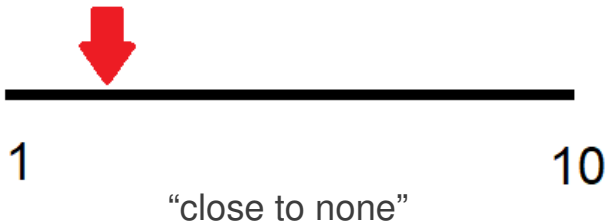
# Main Topics of Study in Math Ed Courses?

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The First Year Mathematics Repository could help to standardize the curriculum for “Math for Teachers” courses, which **seems to vary greatly** from university to university.

# Collaboration Between Math Faculty and Faculty of Education

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In some Math departments, there are links between the Math Faculty and the Education Faculty, at most others there is no strong relationship. It may depend upon whether the Math course is restricted to education students. At some Universities there is a complicated history that plays a role. At others where there are strong collaborations, they are rich and productive.

# Math Anxiety and Education Student Failure Rates in Math Service Courses

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Dealing with math anxiety can be done through

- ▶ an open discussion and acknowledgement of the common issue,
- ▶ creating more/multiple low stakes and scaffolded learning and assessment opportunities,
- ▶ clear expectations and learning objectives, and
- ▶ building relationships between instructor and student (TRUST and CARE) and among groups of students.

The group discussion found that **failure rates for Education students vary by university, but are similar to fail rates in other math service courses.** However, this discussion was based on a small sample size.

# Little Consensus in Math Service Course Offerings to Pre-service Teachers

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- ▶ At some universities, the Education degree is concurrent, while at others it is consecutive. This adds to difficulties in course offerings.
- ▶ At some universities course is taught by dedicated person(s), while at others it is taught by sessional instructors and graduate students.
- ▶ Some universities offer math courses for elementary/middle grade teachers; others offer courses specifically for secondary math teachers. At most universities, secondary level student teachers take math courses with the general undergraduate population. Elementary level math teacher courses usually have restricted enrolment.

# Top Issues in Math Service Course Offerings to Pre-service Teachers

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- ▶ Weak math background of many in-coming Education students.
- ▶ No consensus about math knowledge background/high school pre-requisites required to enter math teacher degree programs.
- ▶ Lack of standardization in math service courses and math teacher degree programs of study across Canada.
- ▶ The disconnect between Mathematicians and Educators.
- ▶ Varying provincial requirements for accreditation of math teachers.
- ▶ Varying provincial mathematics curriculum guides.
- ▶ The variety of university instructors who are assigned to teach the course (sessionals, grad students, post-docs, permanent faculty).

All of these issues **affect the topics and breath of coverage** in math service courses across Canada offered to pre-service teachers.

# Authentic teaching and learning in calculus for life sciences



# Authentic teaching and learning in calculus for life sciences

## Outline

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## Outline

- A summary of our discussion

# Authentic teaching and learning in calculus for life sciences

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- A summary of our discussion
- A proposal

# Authentic teaching and learning in calculus for life sciences

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- A summary of our discussion
- A proposal
- How it looks in practice

# Authentic teaching and learning in calculus for life sciences

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- We teach a wide range of courses (~70 students in 2 sections, to ~1000 students in 9 sections) at institutions where “calculus for life sciences” means a lot of things (an easy terminal course, to a difficult “pre-med” course)

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- One belief we hold in common: good teaching is *authentic*, both from the point of view of the instructor and from the point of view of the students
- *e.g.* pathological functions are inauthentic to students; but without pathological functions, the MVT is inauthentic to instructors



# Authentic teaching and learning in calculus for life sciences

## Summary

- So why teach the MVT?

# Authentic teaching and learning in calculus for life sciences

## Summary

procedural

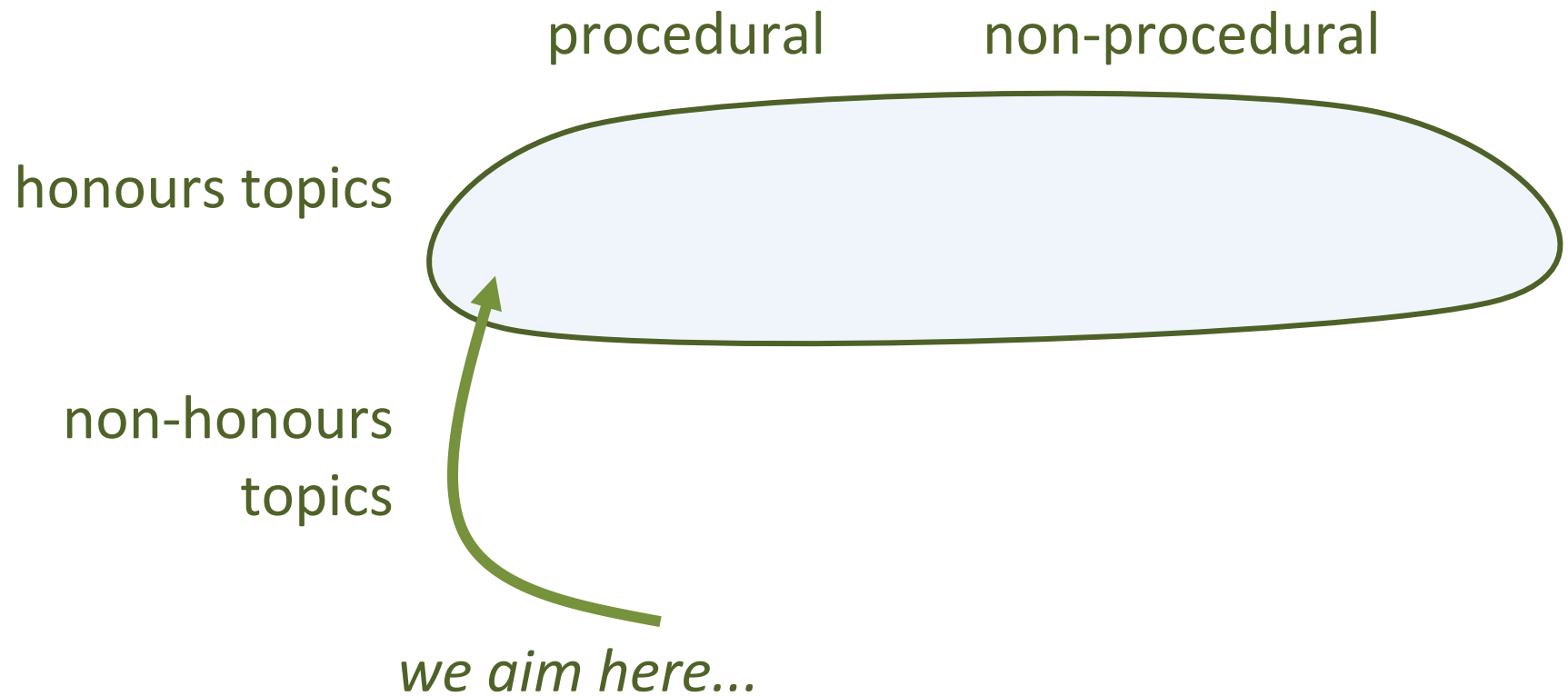
non-procedural

honours topics

non-honours  
topics

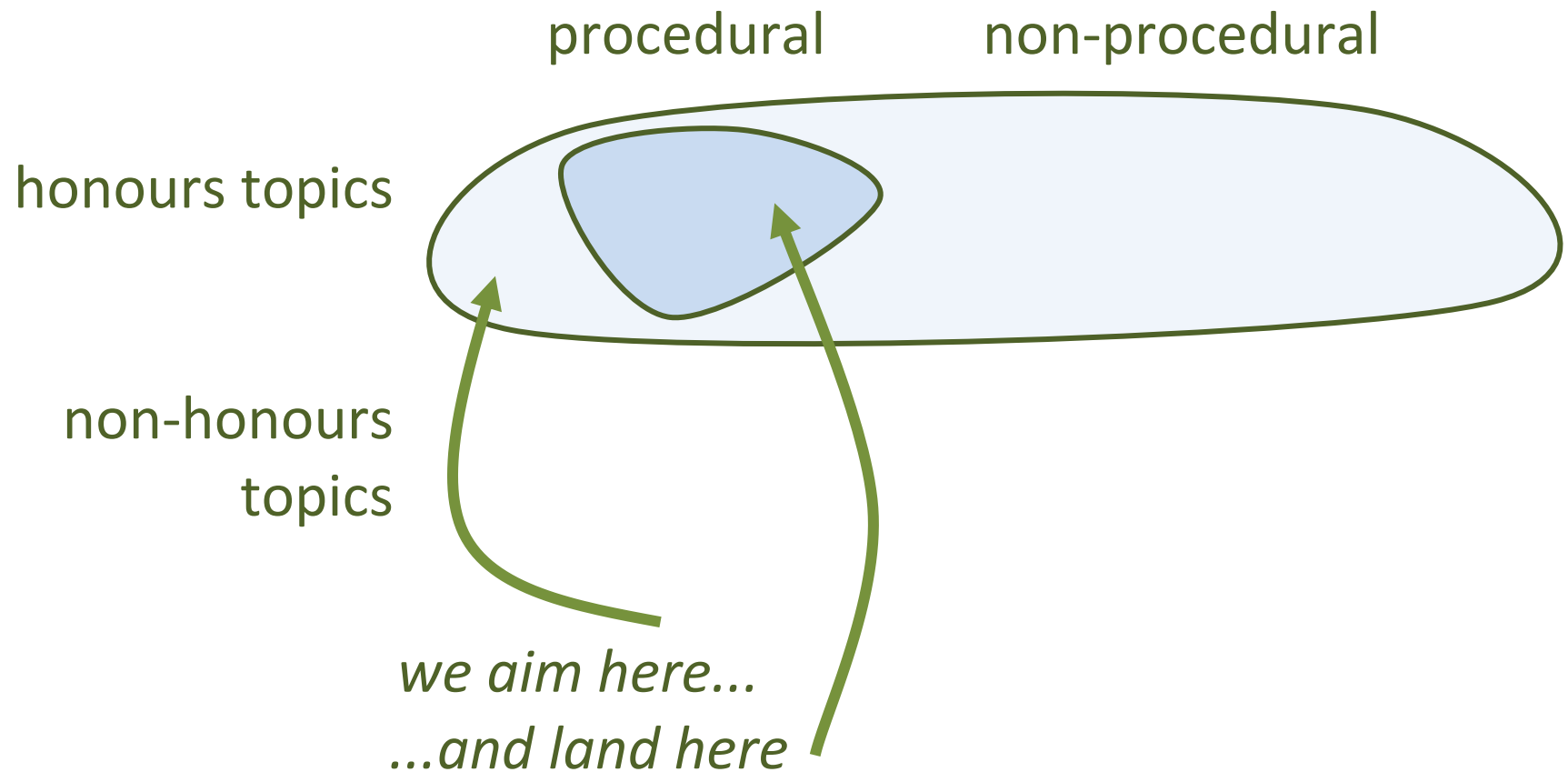
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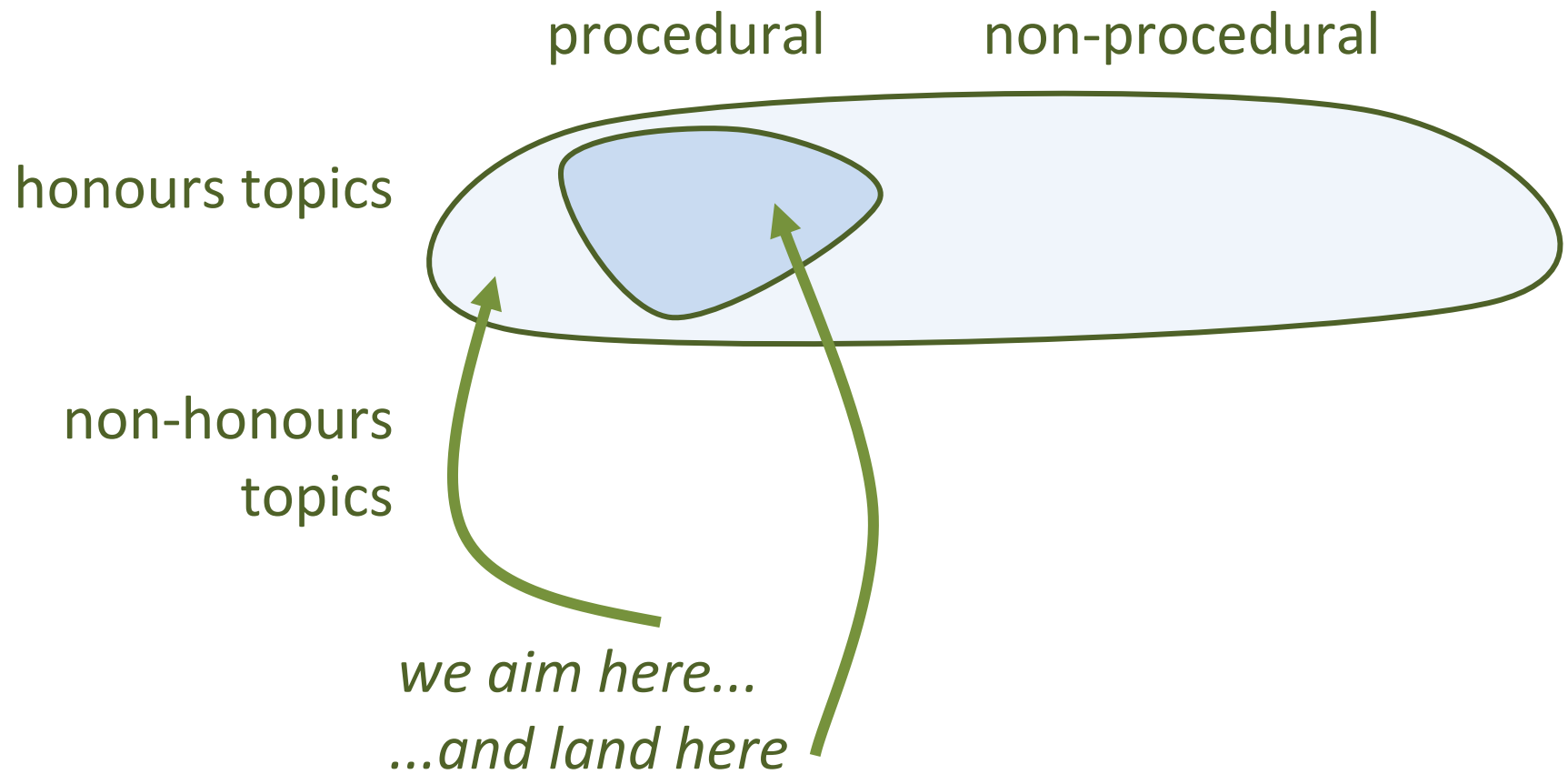
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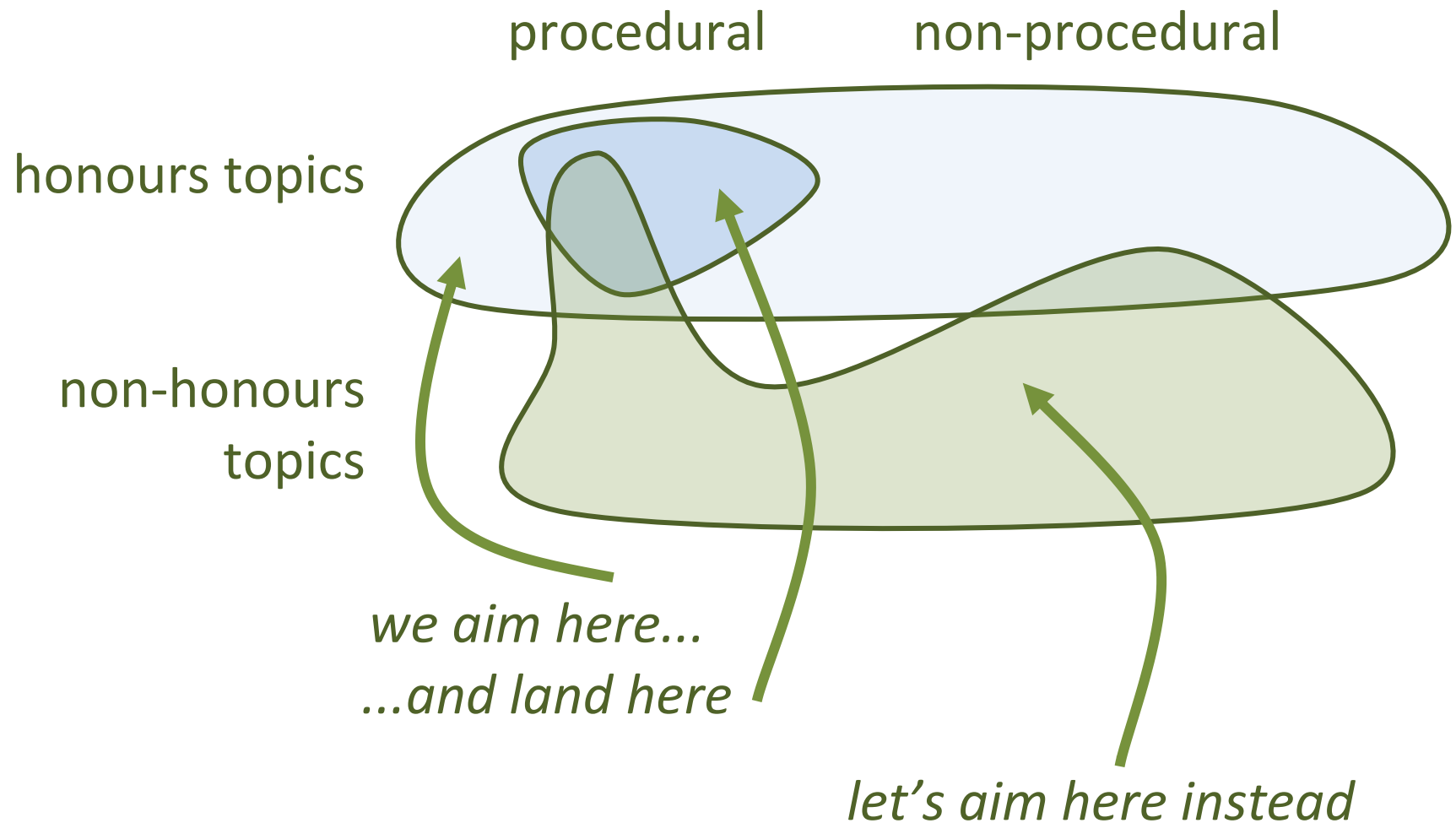
# Authentic teaching and learning in calculus for life sciences

## Proposal



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# Authentic teaching and learning in calculus for life sciences

## Proposal

- What can we give up or reduce?

# Authentic teaching and learning in calculus for life sciences

## Proposal

- What can we give up or reduce?

Pathological functions

$\delta$ - $\epsilon$  and other formal proofs

Rigorous treatments of continuity, differentiability, IVT,  
EVT, MVT

Technical treatments of limits

Standard related rates and optimization problems

Graphing problems with no foothold in science



# Authentic teaching and learning in calculus for life sciences

## Proposal

- What can we gain?

# Authentic teaching and learning in calculus for life sciences

## Proposal

- What can we gain?

  - Introductory DEs, discrete dynamical systems, probability

  - Interpreting results in context

  - Model evaluation (is it suitable? economical?)

  - Comfort with approximations and back-of-the-envelope calculations

  - Deeper, “stickier” understanding of functions and rates of change

  - Scientific skepticism

# Authentic teaching and learning in calculus for life sciences

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- What about instructors' capabilities?

*See below....*

# Authentic teaching and learning in calculus for life sciences

**How it looks in practice**

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# Authentic teaching and learning in calculus for life sciences

## How it looks in practice

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- Instructors meet weekly to discuss these materials, as well as sticking points, modes of exposition, etc.
- Note: these materials exist!

# Authentic teaching and learning in calculus for life sciences

**How it looks in practice**



# Authentic teaching and learning in calculus for life sciences

## How it looks in practice

- Here's an example of a rich, authentic question



# Examples

Kseniya Garaschuk

February 10, 2019

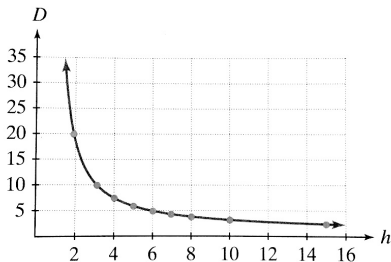
## Applications: crows and whelks

Crows feed on whelks by flying up and dropping the whelks on a hard surface to break them. Biologists have noticed that Northwestern crows consistently drop whelks from about 5 meters. We will attempt to understand why.



## Applications: crows and whelks

Here is the data collected by a Canadian scientist Reto Zach in his famous study on optimal foraging of crows. Zach repeatedly dropped whelks from various height to determine how many drops were required to break the whelk:



*Data Source:* Reto Zach, 1979. Shell Dropping: Decision-Making and Optimal Foraging in Northwestern Crows. *Behaviour*, 68, pp. 106–117.

## Applications: crows and whelks

A best-fitting curve relating the number of drops  $D$  to the height  $h$  (in meters) is given by

$$D(h) = 1 + \frac{20.4}{h - 0.84}, \quad h > 0.84.$$

- 1 Find  $D'(h)$ .

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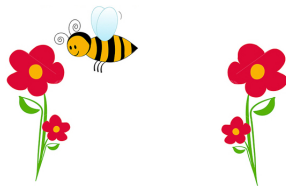
- 1 Find  $D'(h)$ .
- 2 Find  $D'(4)$  and interpret this quantity.
- 3 Find the optimal height from which the crow should drop the whelk to minimize the amount of work required to break it.



# Optimal foraging

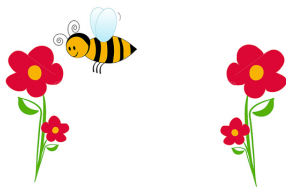
Consider a honey bee collecting nectar:  
it finds a flower, drinks the nectar, it gets  
depleted, the bee has to find another flower.  
Travel takes time.

What does the bee want?



# Optimal foraging

Consider a honey bee collecting nectar:  
it finds a flower, drinks the nectar, it gets  
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Travel takes time.



What does the bee want?

Let  $f(t)$  be the amount of nectar collected by a bee that stays on the flower for time  $t$ .

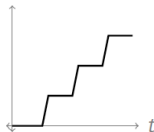
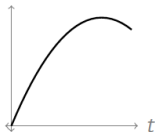
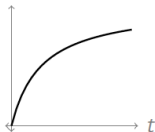
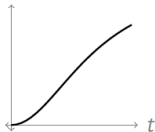
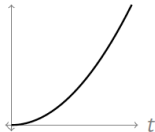
What do you expect  $f(t)$  to look like?

## Which of the graphs matches which description of $f(t)$ ?

A: Collection goes well at first but slows down as the nectar is depleted.

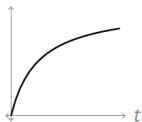
B: It is initially hard to find nectar but gets easier with time. Eventually, there is none left to collect.

C: Initially, you collect some nectar but the humans figure out what you are doing and start stealing from you.



## Bees' objective function

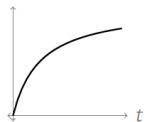
We will consider the first scenario: high collection rate at first, nectar being depleted until there is none left.



Let  $r(t)$  be the rate of nectar collection after  $t$  minutes on the flower. Suppose it takes 4 seconds to get to a new flower.

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We will consider the first scenario: high collection rate at first, nectar being depleted until there is none left.



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$$r(t) = \frac{\text{nectar per visit}}{\text{total time per visit}} = \frac{\text{nectar per visit}}{\text{time on flower} + \text{travel time}} = \dots$$

$$\text{For your convenience, } r(t) = \frac{t}{(t+1)(t+4)} \text{ and } r'(t) = \frac{4-t^2}{(t+1)^2(t+4)^2}.$$