TEACHING MODELING IN FIRST YEAR
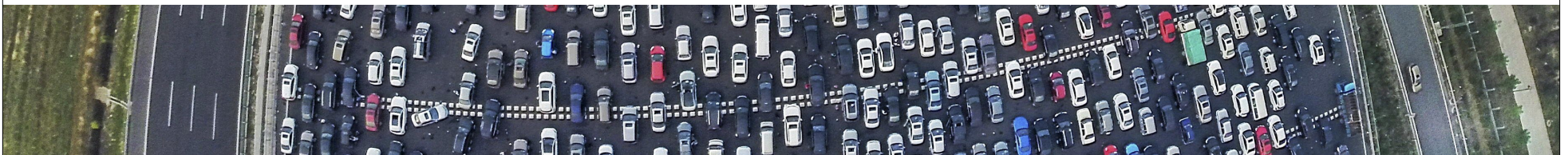
Yvan Saint-Aubin

« Oser, le progrès est à ce prix.

-Victor Hugo

WHY ? (THE GENESIS)

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- *external review committee report (program evaluation):*

  We recommend to inform students, especially those in pure and applied math,
  of possible jobs waiting for them after their degree. This would allow them to realize that
  there are more than the two traditional paths (teacher in cegeps or university professor).
  That seems a simple investment that could be really fruitful.

- *reshaping the BSc curriculum:*

  the first committee was disbanded… But MAT 1460 was created just before!

GESTATION

• *GAIMME report (Guidelines for assessment & instruction in mathematical modeling education) funded by Society for Industrial and Applied Mathematics (SIAM) and COMAP (Consortium for Mathematics and its Applications)*

Chapter 2 Prekindergarden through Grade 8
Chapter 3 High school
Chapter 4 Undergraduate level

GAIMME

GUIDELINES FOR ASSESSMENT & INSTRUCTION
IN MATHEMATICAL MODELING EDUCATION

CONSORTIUM FOR MATHEMATICS AND
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- *learning outcomes:*

  - to transform a real life problem into a well-stated problem;
  - to state hypotheses and justify them;
  - to identify mathematical quantities/objects relevant to the problem;
  - to gather scientifically-sound data relevant to the problem;
  - to cast the well-stated problem into equations/algorithms;
  - to work in group;
  - to report the results (by writing or orally).

Stephanie Peacock (U. Alberta; now at U. Calgary)

THE CONTEXT, THE STEPS OF CREATING A NEW COURSE

- *mat 3450 : modélisation mathématique*

    elective in BSc, pure & applied math
    elective in B en éduc., future high-school teachers

    taught the traditional way: here are mathematical tools that you've learned, let me show you
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- *creation of a course = a departmental meeting*

  colleagues saw obstacles, many obstacles ("Nobody has ever given such a course!" "If we found a colleague to give it, what will happen when he/she leaves?" "There'll be too many students to insure sufficient feedback!" etc., etc., etc.)

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    colleague to give it, what will happen when he/she leaves?" "There'll be too many students to
    insure sufficient feedback!" etc., etc., etc.)

    the students were ecstatic (they actually made it happen!)
        "the most important event of my eight years at DMS!"
        "will MSc and PhD students be allowed to register?"

HELP!

- *GAIMME*

  for the overall approach, for its appendices on ressources, for ideas of projects, for its evaluation tools, …

GAIMME

GUIDELINES FOR ASSESSMENT & INSTRUCTION
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HELP!

- *Math Modeling, Getting started & getting solutions*

  Written by KM Bliss, FR Fowler, BJ Galluzzo, three of the authors of GAIMME
  Aimed at last-year high-school students
  One chapter per step of the modeling process
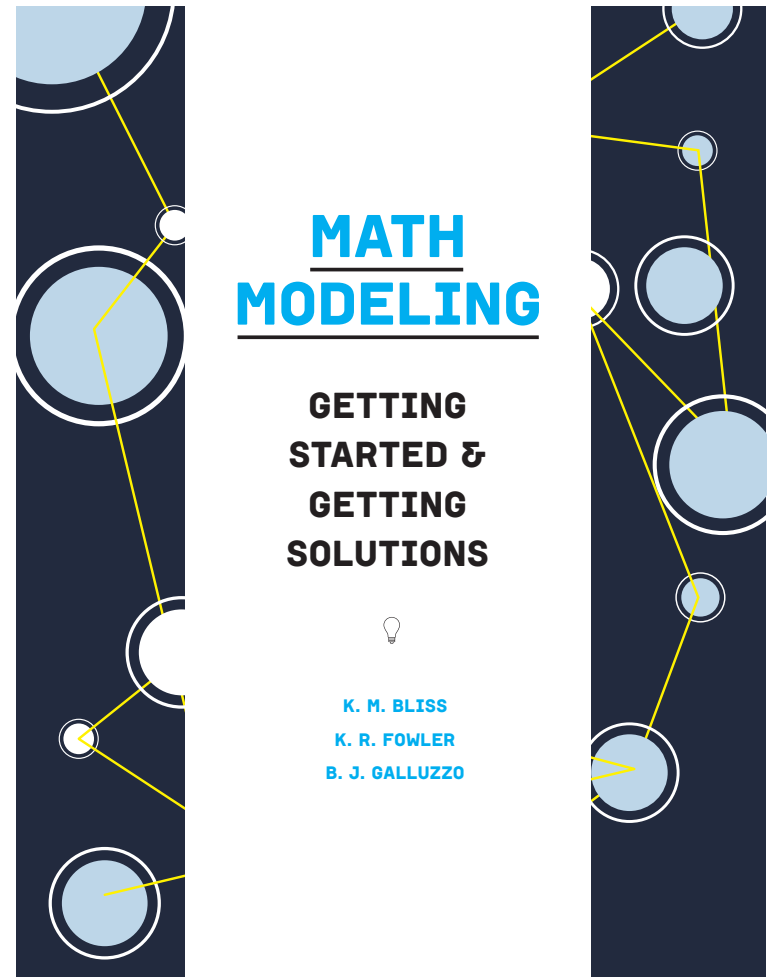 (defining the problem statement,
  making assumptions,
  defining variables,
  building solutions,
  analysis and model assessment,
  putting it all together)

MATH MODELING

GETTING STARTED & GETTING SOLUTIONS

K. M. BLISS
K. R. FOWLER
B. J. GALLUZZO

HELP!

- *ressources on the web*

  Mathematical Contest in Modeling
  Cornell Mathematical Contest in Modeling
  MathModels.org
  …

LET'S DO IT!

"If you're not failing every now and then, it's a sign you're not doing anything very innovative

-Woody Allen

THE ELEVATORS

Forward from HR:

De: CEO <theCEO@theBigCompany.ca>
Objet: they're still late !?@&!
Date: 26 avril 2019 à 21:41:35 UTC−4
À: Human Resources <hr@theBigCompany.ca>
Cc: CEO <theCEO@theBigCompany.ca>

Hey Shophika!

I still get complaints about staff being late, some by 15 minutes.
With the staff we have, that's about one salary lost.
Again the bottleneck of the elevators seems the problem.
Can you suggest solutions?

Thanks, the CEO

STEP A : WHAT DOES THE QUESTION MEAN?

- *make the question precise, bring it into a "mathematical form"*

  "mathematical form" means "choosing the mathematical object" best suited for mathematics
  - a number
  - a geometric form
  - a graph
  - an algorithm
  - …

STEP A : WHAT DOES THE QUESTION MEAN?

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  - a number
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  - an algorithm
  - ...

- *let's pause for a first task*

  - elect a spokesperson;
  - answer the question: what mathematical object would you use
      to convince the CEO you've solved or improved on the problem?
  - be prepared to report in a few minutes.

MY "MATHEMATICAL QUESTION"   =   MY DEPENDENT VARIABLE

Let R be the sum of minutes by which every employee is late.

   Employees that are on time count for 0 min.

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Question: how can R be minimized?

STEP B : BUILDING A MIND MAP

....

- *which "solutions" you want to explore, what are the determining factors, parameters, what information you will need, …*

  this is the first brainstorming
    - stay open minded
    - don't be too critical at this point
    - group elements of same nature together, organize elements
    - keep the time constraint in mind
    - …

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  - …

- *let's pause for the second task*

  - explore possible solutions
  - identify key-elements, data, parameters, …
  - gather info into appropriate subsets
  - be prepared to report

AN EXAMPLE OF A MIND MAP

Ne s'applique qu'à ceux qui ont peu d'étages à gravir et donc pas ceux des 30-33e étages

Utiliser plus souvent les escaliers

Promouvoir l'utilisation des escaliers
- Règlementations
- Primes ou avantages

Réaménagement des escaliers
- Escaliers roulants pour les premiers étages
- Redécorer les escaliers

Arriver d'avance

Prévoir le temps nécessaire pour contrer la congestion dans les ascenseurs

Comportement des employés

Répartir les heures d'arrivée en décalant les quarts de travail
- Employés mécontents du changement d'horaire
- Selon l'étage où se situe le bureau de l'employé
- Selon les disponibilités et les préférences des employés

Longueur d'écart entre les plages horaires
- Minimiser la congestion
- Minimiser l'écart entre les heures d'arrivée et les heures «conventionnelles» de bureau

Respecter la capacité des ascenseurs
- Ne pas dépasser la capacité maximale — Éviter les retards causés par des problèmes mécaniques
- Maximiser l'utilisation de la capacité de l'ascenseur — Ne pas laisser l'ascenseur monter vide ou presque vide

Minimiser le nombre de retards à cause de la congestion dans les ascenseurs

Se regrouper avec des gens qui travaillent au même étage
- Diminuer le nombre d'arrêts des ascenseurs et donc minimiser le temps d'attente
- Réticence des employés à respecter les groupes et les ascenseurs assignés

Comportement des ascenseurs

Augmenter la vitesse ou la capacité, ou ajouter des ascenseurs — Trop coûteux et mis de côté dans l'énoncé

Catégoriser les ascenseurs
- Certains ascenseurs arrêtent seulement à certains étages — Réduire au maximum les retards causés par les nombreux arrêts
- Certains ascenseurs sont uniquement pour les employés
- Aux heures critiques : certains ascenseurs ne font pas d'arrêt en descendant

STEP C : MAKING ASSUMPTIONS

- *concentrate on a few parameters and variables, state hypotheses*

  again, what are the key-factors in the problem:
    - find data when you can, check reliability;
    - if non-available, make reasonable assumptions
    - …

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- *some examples:*

  - number of piano tuners in Edmonton? *(Fermi's problem)*
  - how much sand the City of Edmonton needs for its roads during winter:
      linear km of roads/streets/highways in Edmonton?
  - skating season is shortening in Canada:
      what are the key-factors determining its length?

STEP D : DISTINGUISH "PARAMETERS", "VARIABLES"

- *parameters are "fixed" data, variables can vary!*
  - distinction between dependent and independent variables
  - the choice of question at Step A should determine the *dependent* variable
  - use units as safeguards against "stupid" mistakes
  - …

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- *problem of the elevators better defined*

  - the BigCompany occupies floors 30 to 33 of Place Ville-Marie;
  - the personnel is distributed:
    350 persons on each of floors 30 and 31
    250 on floor 32
    150 on floor 33

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- *I've started Step D for "the BigCompany"*

  (Place Ville-Marie in Montreal was designed in 1962 by Henry Cobb, one of the funding partners of Pei Cobb Freed & Partners)

PARAMETERS, DATA · INDEPENDENT VARIABLES

- *loading time of people at ground floor = 20 sec*

- *speed of an uninterrupted course = 1.5 floors / sec*

- *stop time at a floor = 7 sec / floor*

- *number of elevators serving floors 30 to 33 = 8*
   (these elevators serve floors 23 to 33 (11 floors))

- *maximal capacity = 25 persons*

   - Δt = *time of flight (including stops) from ground to 33rd floor and back*

   - n = *# of elevator flights to bring the staff of the 11 floors*

   - d = *mean # of persons / floors*

   - N = *# of persons working on floors 23 to 33*

   - t = *total time to bring the BigCompany staff to their floor*

MY HYPOTHESES

- *personnel that should start at time t arrive uniformly in the interval [t–30, t–5];*

- *first arrived, first served*

- *during morning rush hour, elevators don't stop on the way down*

- *elevators stop only at half of the floors that they serve;*

- *elevator failures are neglected;*

- *mean # of persons per floor = mean # of persons per floor of the BigCompany*

- *elevators are filled, in average, to 80% of their capacity*

STEP E : CONSTRUCT A MODEL, A FORMULA, A GRAPH?

- *time to tie variables in a mathematical model to answer the "mathematical" question formulated at Step A*

  this might mean
  - write down equations
  - construct a graph
  - analyze a geometric figure
  - do a statistical analysis
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- *attention : the mathematical tools might changed during the course and they might change decisions made and assumptions stated earlier*

STEP F : ANALYSIS OF THE MODEL

• *analysis of the model solutions (= solve the equations, do the statistical analysis, check the output of the algorithm, …)*

does the model
- behaves as expected?
  (order of magnitude is reasonable, sign of answer is right, so are the units?)
- is it robust to change in the parameters (especially those I had to guess)?
- can it be matched to existing data?
- strengths, weaknesses, improvements?
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• attention : this analysis usually forces you to reconsider many of previous steps!

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Mean # of persons per floor for the BigCompany

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Time *Δt* of one flight

Δt = (loading time on ground floor) + (time of flight ground -> 33)

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each trip with 80% x 25 = 20 persons and thus *19 trips will be needed*.

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t = 19 x 106 sec / (60 sec / min) = 33 minutes

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So what's wrong?!

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STEP G : COMMUNICATE THE RESULTS

- *time to report the results*

  could include
    - a detailed report (between 15 to 30 pages)
    - AND/OR an executive summary for the CEO of the BigCompany (1 or 2 pages)
    - AND/OR oral presentation
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- *attention : most students didn't know how to write*
     but, surprise!!! when you teach them, they learn!

WHERE ARE WE
AFTER TWO YEARS ?

« Animals are such agreeable friends – they ask no questions; they pass no criticisms.

-George Eliot

H18 & H19 : TWO EXPERIENCES

- *two profs (Anne Bourlioux and I), one TA & approx. 30 students (work in teams of 4, teams are formed by profs., changed each week or each project)*

- *about 5 weeks to go through Getting Started ; 8 weeks for 3 projects*

- *the first 5 weeks: many, many, many problems (gas-stations, the pink subway line of our new mayoress, pandemics, bottleneck on highways, the parachute path of Felix Baumgartner, prey & predator)*

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The three "big" projetcts (all the steps have to be done, teams of 4, changed after each project). In 2019

- *project 1: how many primary-school kids will there be on Montreal Island for the coming 30 years?*

- *project 2: best strategy for overbooking flights?*

- *project 3: deep impact: various questions around the fall of a 500-m asteroid?*

REACTIONS: THE COLLEAGUES

- *the senior colleagues*

    "they learn no new mathematics in this class!"
    "but there is no mathematics"
    "ah! that's what engineers do"
    "there is so much stuff to cover during the BSc, you really think we have time for that?"

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- *the young ones*

  "wow, cool!"
  "how do you think my phone computes the stairs I've climbed in a day?!"
  "how does my stationary bike compute the calories I burn?"
  … more ideas of projects (some absolutely wild, others intriguing)

REACTIONS: THE STUDENTS

- *students after the first term*

    "cool!"

    "too much time on Getting Started"

    "while covering Getting Started, we should go through one problem from beginning to end"

    "this modeling course doesn't arrive at the right time in the program"

    "one member of my team did nothing!"

    "too much work, the deadline of projects are really stressful!"

    "I came here exactly to do that!"

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- *students in 2nd and 3rd year, grad students*

    "could we come? we would sit in the back…"

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- *two students of the first year funded a "Modeling Club"*

  two teams participated in the 2019 Mathematical Contest in Modeling;
  one got "successful participants" and the other "honorable mention" (top 25%!)

REACTIONS: THE TWO PROFESSORS

- *Anne and I*

    "phew…"

    first year was really scary and an enormous amount of work

    teams assembled by profs (not a problem!)

    enthusiastic atmosphere

    no snubbing between (pure and applied) and (statistics)

    programming is not mastered by all

    lots of work/stress/hesitation in choosing the projects

    how to make sure everybody really participate

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  how to make sure everybody really participate

- *I*

  I managed to find two internships in a high-tech company
  hopefully young colleagues will take it over in the spirit of GAIMME
  how to better choose examples during the time we go over Getting Started

LAST QUESTIONS

should there be such a course (along GAIMME)?

when should it be in the curriculum?

should there be more than one such a course?

DURING THE QUESTION PERIOD

- ★ web site of the course: <u>dms.umontreal.ca/~mat1460</u>

- ★ GAIMME: www.siam.org/reports/gaimme.php

- ★ Getting Started: m3challenge.siam.org/sites/default/files/uploads/siam-guidebook-final-press.pdf

- ★ questions of previous Mathematical Contests in Modeling: <u>www.mathmodels.org/contests.html</u>