Saturday – Group 1

We think that our First Year Calculus students form a heterogeneous group. They have different backgrounds, some are highly motivated and even self-driven, and many are confused, disoriented, and, in general, poorly prepared for the transition from high-school to tertiary school, both mathematically and psychologically. Also, we think that most students are different from us, educators, in many ways, including that they have had more exposure to ongoing technological advancements. Taking all these factors into account, it is clear that students need our support to adjust to the new environment, and that we need to update our teaching on a regular basis; for instance, some of us mentioned we may consider to be less rigid, at least initially, in regard to deadlines, and many of us opined that we must learn more about technology that can be implemented into our practices. In my opinion, there was one important aspect that was not fully addressed in the discussion: "(e)ven if it were possible for teachers to accommodate every student's limitation at any point during the school day, their assistance could undermine the most important aspect of this learning - a student's development of a capability to self-regulate" (on page 65 in [1]).

Among the reasons why mathematics is taught to many students, we discussed the following: first, it is a requirement for subsequent courses in many fields, second, as individuals learn mathematics they develop logical reasoning, a skill in very high demand, and third, that is how is has been done traditionally. Some of us even asserted that, like it or not, one reason for teaching mathematics to many of the students doing a university degree is that it is a filter. Among further comments, there was the importance of aligning the reasons why we are teaching mathematics with the students' needs. For instance, if our students are math users, and not math makers, maybe not covering complex proofs is a good idea.

Now, whether we *should* or *should not* cover prerequisite material or instruct students to write mathematics, is not an easy question. However, we all agreed on the fact that quite often we review basics or train students in writing, before starting teaching the actual content or assigning them tasks, correspondingly. Even though we all feel we have a moral and professional obligation to do so, we think that this is something that should be done in collaboration with school teachers. We know they may be subject to even more constraints than we are, but we cannot carry all that weight alone.

[1] Zimmerman, B. J. (2002). "Becoming a Self-Regulated Learner: An Overview". Theory Into Practice, 41(2), pp. 64-70.

FYMSIC 2019 - Day 2 - Discussion Summary

Working Group 6

May 4, 2019

Our focus was course content. Theses were our leading questions:

1. The future of calculus?
2. Is statistics a new calculus?
3. Is mathematical modelling a new calculus?
4. Discrete mathematics for all?
5. Does geometry have a future?
6. Mathematics for future teachers: a band-aid or an implant?

We identified eight themes in our discussion.

1. Motivation: Student motivation is essential to success. A small portion of our students are self-motivated, but often we have to provide motivation through the presentation of our materials. Focusing on motivation in pedagogy is supported by the education research. But how do we motivate? What content do we choose and how do we explain it? We talked about four possibilities.

   - Basic Knowledge: These are building blocks for future essential study.
   - Practical Application: These are useful tools you will need in your future work.
   - Thinking Habits: Mathematics teaches some of the habits of thought thare are most valuable, most useful, or most desired by industry and society.
   - Joy: The study of mathematics is a glorious enterprise in itself.

   We had no consensus on which of these motivations ought to take precedence. In this and future themes, the lack of fundamental consensus is a major barrier to substantial curricular reform.

2. Purpose: Before we can answer the questions of content, we got stuck on more fundamental questions. We must determine the basic purpose of a first-year mathematics before questions of content, because the content needs to reflect that purpose. So why do we teach calculus? Our suggested possible reasons are very similar to the list of motivations: we can teach to provide foundations for future study, to provide tools for practical applications, to train in habits of thoughts, or to celebrate the intrinsic joy of mathematics. Again, there was no consensus.

3. Habits of Thought: One of the purposes of first year math (or any math education) might be to cultivate habits of thought. In this purpose, the specific content of a mathematics curriculum

is secondary to teaching abstract thinking, logical, proof, analysis, numeracy and other general skills and competencies. If this is, in fact, a major goal of mathematics (which was not necessarily agreed upon), how do we do it? We were skeptical that calculus was the best venue for teaching these general skills: perhaps linear algebra, discrete math, geometry, modelling, or some mixture of these topics would be more effective at teaching habits of thoughts. Certainly, the current first-year curriculum does not seemed designed with habits of thought as a central goal.

4. The Most Applicable: One of the purposes of first year math might be to provide practical tools for application, particularly for students who major outside of mathematics (the vast majority of our first-year students). If this is a main purpose, we wondered what the most applicable mathematics subjects were. Calculus has historical prominence here, and for some disciplines it remains the most useful tool. However, as the needs of society and industry change, some of our group was very open to reconsidering the historical privilege of calculus. Cases can be made for discrete math (discretization of physics in quantum mechanics, computing), linear algebra (modelling and computing) or statistics (the ubiquity of data and growth of data science).

5. Tools vs. Problems: Responding to Yvan Saint-Aubin's talk and the GAIMME report, we discussed how mathematical modelling might become a greater part of curriculum. Some new pedagogy suggests starting with problems to motivate the eventual development of mathematical tools. Traditional curriculum typically starts by teaching mathematical tools and then showing their application after they have been mastered. Some of us spoke for the powerful motivational factor of starting with models; others feared alienating and confusing the students with problems they lacked the tools to approach. We had no consensus on this point.

6. Understanding vs. Memorization: We were unified in wanting any new mathematics curriculum to focus on understanding. We echoed points from the plenary talks that deep understanding of mathematical concepts is necessary to both future development in pure math and for applications of mathematics in complicated new situations.

7. Streamed or Unified Courses: Many large universities are moving to streamed service courses in their first year math offerings for non-math majors. Many disciplines that require mathematics are likewise demanding these streamed courses with content specific to their discipline. We saw streamed courses as a good environment for innovation: a streamed calculus courses could morph into a broader introduction to various piece of mathematics (calculus, linear algebra, etc) that were useful for the discipline. However, we worried about two risks. First, internal and external transfer becomes more difficult as streamed courses diverge from each other. Second and more fundamentally, some of us wanted to embrace teaching first year math, even in service courses, for its own sake: as an introduction to the ideas and techniques of mathematics intrinsically. Streaming and the focus on application may not mesh with this intrinsic motivation.

8. No Universal Method: In the various kinds of first year math courses we offer, there are many different needs, purposes and goals. There are also many different classroom setups, enrolment numbers and resources. Finally, there are many different instructors with their own styles and identities. For all of the lack of consensus we had, we recognized that the answers to many of these questions should be specific to the situation. There isn't a simple, one-size-fits-all approach to first year mathematics. Each instructor, course, department and institution needs to answer the fundamental questions first (needs, purpose) and then design curriculum to match.

Theme: “What should we teach?”
WG2: Teaching Objectives
Notes by Laura Broley, on behalf of the group

- **What do we mean by “First-year Math & Stats”?** The group decided that we do not want to restrict ourselves to speaking about the small number of students who are math majors. “First-year Math & Stats” includes any student taking a first-year course in mathematics or statistics.

- **Should we be teaching mathematical thinking OR mathematical techniques?**
  - We seemed to agree that “OR” should be “AND”: we should be teaching both mathematical thinking AND mathematical techniques. The percentages need to be shifted so that mathematical techniques do not dominate. The challenge is striking the right balance.
  - It is possible to teach mathematical techniques in ways that engage students in mathematical thinking (e.g., get them to combine techniques in creative ways; add more variation to the tasks; get students involved in creating/varying the task).
  - But once a type of task is put on an exam, does it not become routine afterwards?
  - Interconnected Observations and Obstacles:
    - What are students learning? Exactly what we assess.
    - Teaching and assessing mathematical technique (in a traditional sense) is straightforward.
    - We say that students don’t remember what is done in a course a year later; we are encouraging that behaviour with our assessments.
- **We seemed to agree: Once we clarify our objectives, we need to carefully design our assessments to address them.**

- **But what are our objectives? Starting a List:**
  - Should we develop a list of transversal skills (i.e., skills that are not specific to particular mathematical content)? For example: writing, communicating, …
  - “I also want my students to understand the forward implication and universal and existential operators.” These can be slipped into anything you teach. They could also be emphasized in a course dedicated to pure logic, ways of thinking, puzzles, games, …
  - Students should also learn how to interpret terms like dependent variable and independent variable, they should learn about the difference between a parameter and a variable, and they should learn what a function is. We have the tools to help students explore these things now (e.g., simple tools like Geogebra).
  - Students need to understand different types of graphs.
  - They should also learn about data and what one does with it.
  - …

- **When we list these objectives, are we imagining creating a whole new course?** Alternatively, we could imagine taking existing courses, developing a different set of objectives, and shooting for a deeper integration of them. We currently barge through concepts so quickly that we don't take time on the most fundamental concepts and the most important objectives. We don't take the time to overcome compartmentalization and tie courses together. So students don't see it. One option would be to shun some things (e.g., exercises, the 8 proofs you've got to know for calculus) to homework and make class all about mathematical thinking.

- **What about proof?**
    - Mathematical thinking means different things: e.g., it could be used to solve applied problems or to engage in mathematical proof. Both should be there.
    - But do we really care about them learning to do a proof? Or do we care about logical thinking? For example, concepts like the forward implication and universal and existential operators don't have to be all about learning proofs.
    - What about *exposing* them to proof? Those who are interested can continue with it in later years. Can "gaining exposure to proof" be an objective?
    - Maybe we should expect them to get it, but be ok if a large portion of them don't. Many of these students will be doing something other than pure mathematics.

- **Should our objectives vary depending on the student?**
    - Maybe there could be a group of hardcore objectives that are expected for every student, and then a group of objectives for different grade levels: to get an A, you need this, to get a B, you need this, and so on.
    - Maybe objectives should vary depending on students' goals. But what if there's 100 different goals in the classroom? It seems we are moving away from specialized calculus streams. If testing needs to be standardized, maybe we could achieve variance in what the students do at home.
    - But how many students know what their goal is in their first year? Should first year be more about breadth? Is our aim to plant seeds, some of which will grow, some of which will not?
    - We might envision a general course: "Mathematical and Computational Thinking for Science".
    - But then: What about engineering? They have their own specific objectives and might think they can do it better. Also: Would the engineers succeed in such a course?
- **How much should other disciplines be involved in setting the objectives? Should *we* be making *all* the decisions?**

- **It seems we are struggling with the notion of "objective".**
    - It may help to come up with a definition. For example, Google distinguishes between *objectives* and *key results*. An objective is a high-level goal, whereas a

key result is a quantifiable metric. If 60% of the key results are met, then that's considered an "A".
- As a teacher, does it make sense to expect a large portion of students to not be able to reach some of the important goals?
- Maybe we want *all* students to be able to grapple with the "stuff" we want to teach, knowing that only some will be able to learn the "stuff" (the word "stuff" stands for more than content).
- After all: If we don't aim high, are we pushing our students hard enough?
- If every student can get every question on an exam, is it a good exam? Even with something like forward implication, there are exercises that could make it routine and those that could emphasize understanding.

• **Once again, we seemed to agree on the importance of assessments: We need to develop better assessments to help our students reach the objectives.**

• **But what are our objectives? Continuing the List:**
  - We should help students learn how to think about things clearly. They need to be able to do that before they can engage in proof. They'll eventually realize that they need to be rigorous.
  - We should teach students the importance of being precise in making definitions. They should be able to come up with an ideal definition, poke holes in it, refine it, and give examples. Memorizing a definition is not an objective.
  - Students should gain the ability to question things, including their own assumptions. This means that they need to be in a space where they can admit that they don't know.
  - We should teach students about mathematical logic.
  - They should get to see and play with beautiful interacting structures and learn about mathematical discovery.
  - We should teach them computational thinking, algorithmic thinking, iteration, recursion, discretization, estimation, and programming. We don't necessarily need to teach them a complicated programming language – it could be Excel.
  - Students should learn how to experiment and classify outcomes (e.g., Are we landing on a steady state?). This is necessary to bridge the gap for science students.
  - Students need to get their hands dirty and apply mathematics.
  - They should be shown that mathematics is used outside of mathematics.
  - We also want them to see the beauty in what we're teaching. They should be able to answer the questions: Why do we care? Why should you have taken this course?
  - We should help them learn to communicate what they're thinking. We may see that they know more than we think!
  - Students should engage in peer learning, groupwork, and collaboration (e.g., we could introduce some peer assessment).

  - We may also want students to have some computational fluency (e.g., be able to differentiate a slightly ugly function by hand). This can be a proxy for gaining confidence in mathematics, for being able to put pen to paper and work it out.
  - We want students to be unafraid to try.
  - We want them to gain confidence, persistence, and resilience.
  - We also want them to be able to actively consider different tools to get from A to B, to carefully think about what should be done, and to critically assess a solution.
  - Can we teach them to be able to look ahead and have a gut feeling for where they need to go with a solution to a problem?
  - …
- **We will likely need more than "First-year Math & Stats" to reach many of these objectives. But we can certainly support students in moving in this direction!**

What should we teach?

Focus: Students

Overarching Learning Outcomes/Competencies

One participant in this working group "crossed the quad" to ask faculty in serviced departments (eng, life sciences, business) what knowledge they expect math students to bring with them. Surprisingly, these external departments exclusively identified habits of mind as desired skills. This is one instance of a larger wave of changing values developing in contemporary mathematics education, supported by employers and educators: there is a changing of values, away from skills of the formal mathematical system (integration by cylindrical shells, Riemann sums, Taylor Series, etc.) and toward an explicit education which integrates these skills with habits of mind. Habits of mind include the following:

- critical thinking
- group work/effective collaboration
- logic
- problem solving
- effective learning habits
- independent learning
- managing information
- communicating
- global citizenship
- resilience

Changing The Culture

How can we incorporate habits of mind into mathematics courses? Explicitly. Our discussion gravitated toward changing the culture, both with students and with our colleagues. The following points were made.

Convincing Students	Convincing Educators
• professors are not there to tell students how to calculate • learning is <u>hard</u> and worthwhile • shift focus from content knowledge acquisition and application to thinking processes • mathematics is simultaneously a formal system and a process of thinking	• mathematics is simultaneously a formal system and a process of thinking • be explicit and transparent about expectations (learning outcomes, thinking processes, assessment)

Implementation and Assessment of Habits of Mind within Mathematics

There are lots of ideas for how to implement (e.g. Active Learning, Problem Based Learning) assess habits of mind. It takes more resources, but <u>it is worth it</u>.