MATH 1200 A, Questions for Logic and Indirect Proof

1. *From Rosen, Discrete Mathematics and its Applications:*

   When three professors are seated in a restaurant, the hostess asks them: "Does everyone want coffee?" The first professor says: "I do not know." The second professor then says: "I do not know." Finally, the third professor says: "No, not everyone wants coffee." The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

2. Here is a similar problem

   Three men – A, B, and C – are blindfolded and told that either a red or green hat will be placed on the head of each. The blindfolds are then removed, so each of the three can see the colours of the hats of the other two, but neither can see the colour of his own. Actually, all three are given green hats. The three are then asked to raise a hand if they see at least one greet hat, so all three raise their hand. Then they are asked to lower their hand if they know what they have. After some pause the cleverest of the three lowers his hand. How does he know the colour of his hat?

3. Let n be an integer. Prove each of the following. Clearly distinguish between the two statements and between the means used to verify that they are correct.

   (a) If n is even then n^2 is even.

   (b) If n^2 is even then n is even.

4. Let m and n be integers. Prove that if $3m + 5n$ is even then both m and n are even or both m and n are odd.

5. A real number x is *rational* provided there exist integers m and n such that $x = \dfrac{m}{n}$. A real number is *irrational* if it is not rational. Examples of irrational numbers are $\sqrt{2}$, π, e. Let x and y be real numbers.

   (a) Prove that if x and y are rational then $x + y$ is rational.

   (b) Prove that if $x + y$ is irrational then x is irrational or y is irrational.

   (c) If $x + y$ is rational does it follow that x or y is rational? Answer yes or no then justify your answer.