Proofs and Proving

Determine whether each of the following are valid proofs of the statements given. Be specific. Where appropriate, formulate correct proofs.

1. **Statement:** Let m be an integer. If m^2 is even, then m is even.
   Proposed "Proof": Assume m is even. Then $m = 2k$ for some integer k. Thus, $m^2 = (2k)^2$, or $m^2 = 2(2k^2)$, which is even. Therefore, if m^2 is even, then m is even.

2. **Statement:** Let m be an integer. If m is even, then m^2 is even.
   Proposed "Proof": Suppose m is not even. Then m is odd. So $m = 2k+1$ for some integer k. Therefore, $m^2 = (2k+1)^2 = (4k^2+4k+1)$ which is odd. Thus, if m is odd, then m^2 is odd. Therefore, if m is even, then m^2 is even.

3. **Statement:** Let x and y be real numbers. If $xy = 0$, then $x = 0$ and $y = 0$.
   Proposed "Proof": There are two cases.
   Case 1: If $x = 0$, then $xy = 0 \cdot y = 0$.
   Case 2: If $y = 0$, then $xy = x \cdot 0 = 0$.
   In either case, $xy = 0$.

4. **Statement:** Let m be an integer. If m^2 is even, then m is even.
   Proposed "Proof": Assume m is not even. Then m is odd. Thus, $m = 2k+1$ for some integer k. Then $m^2 = (2k+1)^2 = 4k^2 + 4k + 1$, which is odd. Thus, if m is not even, then m^2 is not even. Therefore, if m^2 is even, then m is even.

Work on these in groups with the support of the TAs.

You should be able to give a clear explanation of your solutions.