“The” Mathematical Proof Writing Handbook

aka “Dr. Thi Dinh’s Bible”

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What do you have to do to prove the following statements?

(Don't answer that...)

1. For all odd integers n, $n^2 + 2n$ is odd.
2. For any $z \in \mathbb{C}$, if $z = \bar{z}$, then $z \in \mathbb{R}$.
3. There exists a function $f : \mathbb{Z} \to \mathbb{Z}$ so that f is one-to-one, but f is not increasing.
4. Let A be an $m \times n$ matrix and let $\boldsymbol{b} \in \mathbb{R}^m$.
   If $\boldsymbol{x}_0$ is a solution to $A\boldsymbol{x} = \boldsymbol{b}$, then any other solution can be written as $\boldsymbol{x}_0 + \boldsymbol{z}$ for some $\boldsymbol{z} \in \mathrm{null}(A)$.

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Answer: Follow "The" Proof Writing Handbook [1]

“The” Proof Writing Handbook

- Let P and Q be statement variables.
- When needed, suppose that $P = P(x)$ depends on a variable x.
- The symbol “$\forall$” means “for all” or “for any”.
- The symbol “$\exists$” means “there exists” or “for some”.

Type of statement	What we must do to prove that it is true
(1) If P, then Q (2) $\forall\, P$, Q	Suppose that P is true. Prove that Q is true.
(3) $\exists\ x\ P(x)$ such that Q	Choose** x so that $P(x)$ is true. Prove that Q is true.

You **do not need to explain how you find x, nor do you need to construct **all possible** x.

"The" Commandment

Rule #1: To prove that a statement is **false**, you must **write out the negation of the statement and prove that.**

"The" Parables

On cases: To prove "If P_1 or P_2 or P_3, then Q".
Suppose P_1. Prove Q.
Suppose P_2. Prove Q.
Suppose P_3. Prove Q.

On being indirect: To prove P by contradiction:
Suppose "not P." Obtain a contradiction. Conclude P.

"The" Cardinal Sins (Five Common Mistakes to Avoid)

- When proving any of the types of statements (1), (2), or (3):
  1. You cannot suppose that Q is true.
  2. You should not overuse symbols nor violate the rules of grammar.†

  † You **must** write in full sentences and use symbols correctly.
- When proving a statement of the form (2) "$\forall\, P,\ Q$":
  3. You cannot "choose" or exhibit an example in place of a proof.
- When proving a statement of the form (3) "$\exists\ x\ P(x)$ such that Q":
  4. You should not attempt to construct all possible x so that $P(x)$ and Q are true.
- When proving a statement **by contradiction**:
  5. You cannot claim a contradiction has been reached without explanation.††

  †† You **must** clearly identify the contradiction being made by making a statement of the form "P and NOT P, which is a contradiction".

References I

Lauren DeDieu, Jerrod M. Smith, Kimberly Golubeva, and Christian Bagshaw, *A resource bank for writing intensive mathematics courses*, Available at http://proof.ucalgaryblogs.ca, 2021.