“THE” PROOF WRITING HANDBOOK

The basics

- Let P and Q be statement variables.
- When needed, suppose that $P = P(x)$ depends on a variable x.
- The symbol “$\forall$” means “for all” or “for any”.
- The symbol “$\exists$” means “there exists” or “for some”.

Type of statement	**What we must do to prove that it is true**
(1) If P, then Q	Suppose that P is true.
(2) $\forall\, P$, Q	Prove that Q is true.
(3) $\exists\ x\ P(x)$ such that Q	Choose** x so that $P(x)$ is true.
	Prove that Q is true.

You **do not need to explain how you find x, nor do you need to try to construct **all possible** x.

Rule #1

To prove that a statement is **false**, you <u>must</u> **write out the negation of the statement and prove that.**

Five common mistakes that you MUST avoid

- When proving any of the types of statements (1), (2), or (3):
  1. **You cannot:** suppose that Q is true.
  2. **You should not:** overuse symbols nor violate the rules of grammar.†

  † You **must** write in full sentences and use symbols correctly.
- When proving a statement of the form (2) “$\forall\, P$, Q”:
  3. **You cannot:** “choose” or exhibit an example in place of a proof.
- When proving a statement of the form (3) “$\exists\ x\ P(x)$ such that Q”:
  4. **You should not:** attempt to construct all possible x so that $P(x)$ and Q are true.
- When proving a statement **by contradiction** (see below):
  5. **You cannot:** claim a contradiction has been reached without explanation.††

  †† You **must** clearly identify the contradiction being made by making a statement of the form “P and NOT P, which is a contradiction”.

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Cases and Contradiction

Proof by Cases. To prove: "If P_1 or P_2 or P_3, then Q." Use cases.

Proof.

Case 1: Suppose P_1. Prove Q.
Case 2: Suppose P_2. Prove Q.
Case 3: Suppose P_3. Prove Q.

$\square$

Proof by Contradiction. To prove Q **by contradiction**.

Proof.

Suppose NOT Q.
Obtain a contradiction.
Conclude Q. $\square$

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