THE PROOF WRITING HANDBOOK

The basics

• Let \( P \) and \( Q \) be statement variables.
• When needed, suppose that \( P = P(x) \) depends on a variable \( x \).
• The symbol “\( \forall \)” means “for all” or “for any”.
• The symbol “\( \exists \)” means “there exists” or “for some”.

<table>
<thead>
<tr>
<th>Type of statement</th>
<th>What we must do to prove that it is true</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) If ( P ), then ( Q )</td>
<td>Suppose that ( P ) is true.</td>
</tr>
<tr>
<td>(2) ( \forall P, Q )</td>
<td>Prove that ( Q ) is true.</td>
</tr>
<tr>
<td>(3) ( \exists x ) ( P(x) ) such that ( Q )</td>
<td>Choose** ( x ) so that ( P(x) ) is true.</td>
</tr>
<tr>
<td></td>
<td>Prove that ( Q ) is true.</td>
</tr>
</tbody>
</table>

**You do not** need to explain how you find \( x \), nor do you need to try to construct all possible \( x \).

Rule #1

To prove that a statement is false, you must write out the negation of the statement and prove that.

Five common mistakes that you MUST avoid

• When proving any of the types of statements (1), (2), or (3):
  1. **You cannot:** suppose that \( Q \) is true.
  2. **You should not:** overuse symbols nor violate the rules of grammar.†
      † You must write in full sentences and use symbols correctly.

• When proving a statement of the form (2) “\( \forall P, Q \)”:
  3. **You cannot:** “choose” or exhibit an example in place of a proof.

• When proving a statement of the form (3) “\( \exists x \) \( P(x) \) such that \( Q \)”:
  4. **You should not:** attempt to construct all possible \( x \) so that \( P(x) \) and \( Q \) are true.

• When proving a statement by contradiction (see below):
  5. **You cannot:** claim a contradiction has been reached without explanation.††
      †† You must clearly identify the contradiction being made by making a statement of the form “\( P \) and NOT \( P \), which is a contradiction”.

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Proof by Cases. To prove: “If $P_1$ or $P_2$ or $P_3$, then $Q$.” Use cases.

Proof.

Case 1: Suppose $P_1$. Prove $Q$.
Case 2: Suppose $P_2$. Prove $Q$.

Proof by Contradiction. To prove $Q$ by contradiction.

Proof.

Suppose NOT $Q$.
Obtain a contradiction.
Conclude $Q$. 

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