In the beginning...

Let \( P \) and \( Q \) be statement variables. When needed, suppose that \( P = P(x) \) depends on a variable \( x \). The symbol “\( \forall \)” means “for all” or “for any”. The symbol “\( \exists \)” means “there exists”.

<table>
<thead>
<tr>
<th>Type of statement</th>
<th>What we must do to prove that it is true</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) If ( P ), then ( Q )</td>
<td>Suppose that ( P ) is true.</td>
</tr>
<tr>
<td>(2) ( \forall P, Q )</td>
<td>Prove that ( Q ) is true.</td>
</tr>
<tr>
<td>(3) ( \exists x P(x) ) such that ( Q )</td>
<td>Choose** ( x ) so that ( P(x) ) is true. Prove that ( Q ) is true.</td>
</tr>
</tbody>
</table>

**You do not** need to explain how you find \( x \).

The first (and only) commandment

To prove that a statement is false, thou shalt write out the negation of the statement and prove that.

The five cardinal sins

- When proving any of the types of statements (1), (2), or (3):
  1. **Thou shalt not:** suppose that \( Q \) is true.
  2. **Thou shalt not:** overuse symbols and violate the rules of English grammar.†
     † You must write in full sentences and use symbols correctly.
- When proving a statement of the form (2) “\( \forall P, Q \)”:
  3. **Thou shalt not:** “choose” or exhibit an example in place of a proof.
- When proving a statement of the form (3) “\( \exists x P(x) \) such that \( Q \)”:
  4. **Thou shalt not:** attempt to construct all possible \( x \) so that \( P(x) \) and \( Q \) are true.
- When proving a statement by contradiction:
  5. **Thou shalt not:** claim a contradiction has been reached without explanation.††
     †† You must clearly identify the contradiction being made by making a statement of the form “\( P \) and NOT \( P \), which is a contradiction”.

*Date: May 19, 2021.*