

Reinventing Calculus

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The calculus reform movement goes back 40 years. While it introduced new kinds of problems, it forgot to change the “laundry list” structure of the curriculum. The result of this is too often a fragmented unimaginative curriculum that fails to prepare most of our students for the world they are entering. I will explain what I mean by “the structure of the curriculum” and will illustrate this with a couple of examples.

The Laundry List

Calculus I, First Semester

Chapter 1. Rates of Change, Tangent Lines and Differentiation

- 1.1. Newton's Calculus
- 1.2. Leibniz' Calculus of Differentials
- 1.3. The Chain Rule
- 1.4. Trigonometric Functions
- 1.5. Implicit Differentiation and Related Rates

Chapter 2. Theoretical Considerations

- 2.1. Limit Operations
- 2.2. Limits at Infinity
- 2.3. Some Basic Theorems
- 2.4. L'Hôpital's Rule

Chapter 3. Extrema, Concavity and Graphs

- 3.1. Monotonicity and the First Derivative
- 3.2. Optimization
- 3.3. Concavity and the Second Derivative
- 3.4. Graphing Functions
 - Rational Functions
 - Other Sketches

Chapter 4. Integration: A Differential Equations Approach

- 4.1. Antiderivatives
- 4.2. Area
- 4.3 Separation of Variables
- 4.4. The Exponential Function
- 4.5 The Logarithm
- 4.6 Growth and Decay
 - Inhibited Growth

Chapter 5. Integration: The Accumulation Method

- 5.1. The Definite Integral he Fundamental Theorem of the Calculus
- 5.2. Summation and the Definition of Area
- 5.3. Volume
 - Volumes of Revolution
- 5.4. Arc Length
- 5.5. Work
- 5.6 Mass and Moments
 - Centroids

Calculus II, Second Semester

Chapter 6. Transcendental Functions

- 6.1. Inverse Functions
- 6.2. The Inverse Trigonometric Functions
- 6.3 First Order Differential Equations

Chapter 7. Techniques of Integration

- 7.1. Substitution
- 7.2. Integration by Parts
- 7.3. Partial Fractions
- 7.4. Trigonometric Methods

Chapter 8. Indeterminate Forms and Improper Integrals

- 8.1. L'Hôpital's Rule
- 8.2 Other Indeterminate Forms
- 8.3 Improper Integrals: Infinite Intervals
- 8.4 Improper Integrals: Finite Asymptotes

Chapter 9. Sequences and Series

- 9.1. Sequences
- 9.2. Series
- 9.3. Tests for Convergence
- 9.4. Power Series
- 9.5. Taylor Series

Chapter 10 Numerical Methods

- 10.1. Taylor Approximation
- 10.2. Newton's Method
- 10.3. Numerical Integration

Chapter 11. Conics and Polar Coordinates

- 11.1. Quadratic Relations
- 11.2. Eccentricity and Foci
- 11.3. String and Optical Properties of the Conics
- 11.4. Polar Coordinates
- 11.5. Calculus in Polar Corrdinates

Chapter 12. Second Order Linear Differential Equations

- 12.1. Homogeneous Equations
- 12.2. Behavior of the Solutions
- 12.3. Applications
- 12.4. The Inhomogeneous equation

Calculus III, Third Semester

Chapter 13. Vector Algebra

- 13.1 Basic Concepts
- 13. 2. Vectors in the Plane
- 13.3. Vectors in Space
- 13.4. Lines and Planes in Space

Chapter 14. Particles in Motion; Kepler's Laws

- 14.1 Vector Functions
- 14.2 Planar Particle Motion
- 14.3 Particle Motion in Space
- 14.4 Derivation of Kepler's Laws of Planetary Motion from Newton's Laws

Chapter 15. Coordinates and Surfaces

- 15.1 Change of Coordinates in Two Dimensions
- 15.2 Special Coordinate Systems
- 15.3 Surfaces; Graphs and Level Curves
- 15.4 Cylinders and Surfaces of Revolution
- 15.5. Quadric Surfaces

Chapter 16. Differentiable Functions of Several Variables

- 16.1 The Differential and Partial Derivatives
- 16.2 Gradients and Vector Methods
- 16. 3 Theoretical Considerations
- 16.4 Optimization
 - The Method of Lagrange Multipliers

Chapter 17. Multiple Integration

- 17.1 Integration on Planar Regions
- 17.2. Applications
- 17.3. Theoretical Considerations
- 17.4. Integration in Other Coordinates
- 17.5 Triple Interals
 - Integration in Other Coordinates

Chapter 18. Vector Calculus

- 18.1 Vector Fields
- 18.2 Line Integrals and Work
- 18.3 Independence of Path
- 18.4. Green's Theorem in the Plane
- 18.5 Stokes' and Gauss' Theorems in Three Dimensions

Whitehead (1922)

On “necessary antecedence”:

It is impossible to read Hamlet until you can read; and the study of integers must precede the study of fractions. And yet even this firm principle dissolves under scrutiny. The danger of the principle is that it is accepted in one sense, for which it is almost a necessary truth, *and it is applied in another sense for which it is false*. You cannot read Homer before you can read: but many a child, and in ages past many a man, has sailed with Odysseus over the seas of Romance by the help of the spoken word of a mother, or of some wandering bard. The uncritical application of the principle of the necessary antecedence of some subjects to others has, in the hands of dull people with a turn for organization, produced in education the dryness of the Sahara.

