DYNAMICAL SYSTEMS INSTEAD OF CALCULUS

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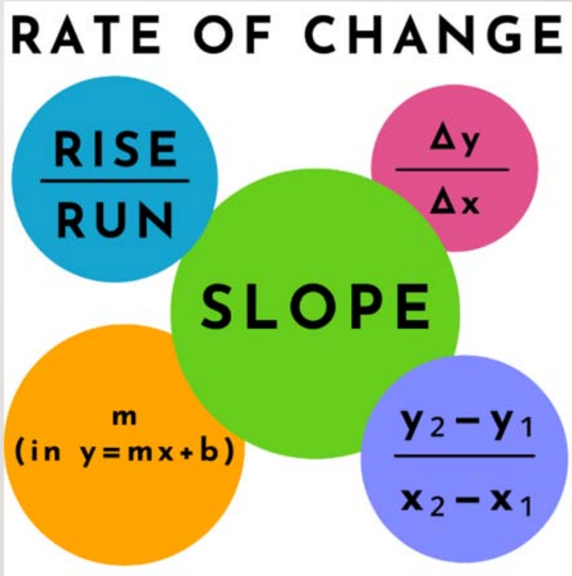
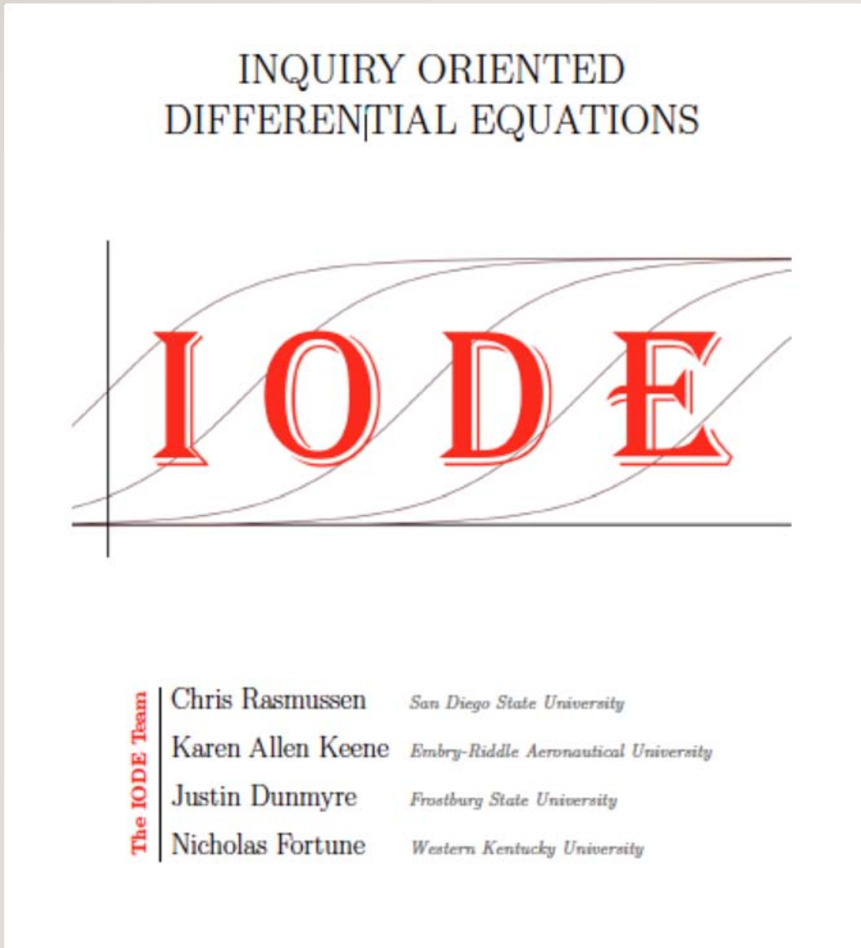
RATE OF CHANGE EQUATIONS AS CENTRAL

- Students have a wealth of experience related to rate of change
  - How fast is some quantity growing or shrinking?
- What about rate of change is needed or that can be developed?
  - dy/dt as △y/△t
  - The quantity dy/dt as an indicator of the steepness of a graph of y vs t
- What about rate of change will be developed?
  - Rate of change as a function
  - Rate of change in some quantity as the means to reveal how the quantity changes
- What is NOT needed or developed?
  - Symbolic differential rules
  - Analytic techniques for finding exact solutions



DYNAMICAL SYSTEMS USING QUALITATIVE, GRAPHICAL, AND NUMERICAL PERSPECTIVES

- Formulate models for a variety of different dynamical systems in biology, physics, and other STEM disciplines.
- Make connections to and extend key high school mathematics ideas and concepts (rate of change, function, function transformation).
- Develop expertise in mathematical practices of modeling, using appropriate tools strategically, making sense of problems and persevere in solving them, and constructing viable arguments and critiquing the reasoning of others.
- Analyze the behavior of solutions to rate of change equations, and their relationship to the contextual significance through graphical, numerical, and qualitative analyses.
- Curriculum inspired by the instructional design theory of Realistic Mathematics Education with design principles Guided Reinvention, Emergent Models, Didactical Phenomenology.



https://iode.wordpress.ncsu.edu/

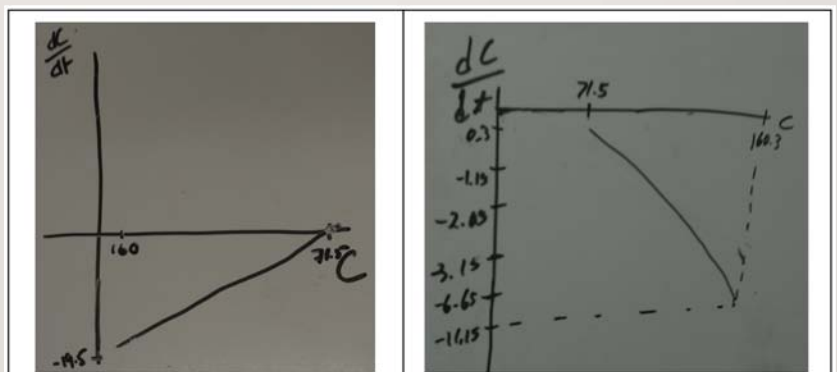
COURSE OUTLINE
(MINUS UNITS THAT REQUIRE CALCULUS)

- Unit 1 – Qualitative and Graphical Approaches (Slope fields get developed here)
- Unit 2 – Numerical Approaches (Euler's method gets developed here)
- Unit 3 – Salty Tank Problems and Linear Rate of Change Equations
- Unit 4 – Autonomous Rate of Change Equations (Phase lines)
- Unit 5 – Modeling with Autonomous Rate of Change Equations
- Unit 6 – The Effect of Varying a Parameter in an Autonomous Rate of Change Equation
- Unit 7 – Introduction to Systems of Non-Linear Autonomous Rate of Change Equations
- Unit 8 – Spring Mass Systems
- Unit 9 – Eigentheory and Linear Systems of Rate of Change Equations

COFFEE COOLING TASK TO MOTIVATE AUTONOMOUS DERIVATIVE GRAPHS

Students are provided with a table of temperature (C) and time (t) data for a cup of cooling coffee. Their task is to figure out a way to use this data to approximate dC/dt, graphically and symbolically.

Time (min)	Temp. (°F)
0	160.3
2	120.4
4	98.1
6	84.8
8	78.5
10	74.4
12	72.1
14	71.5



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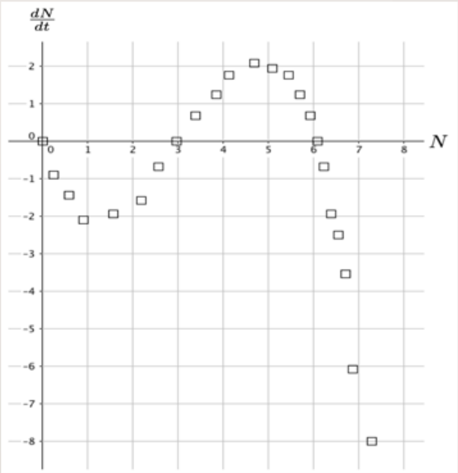
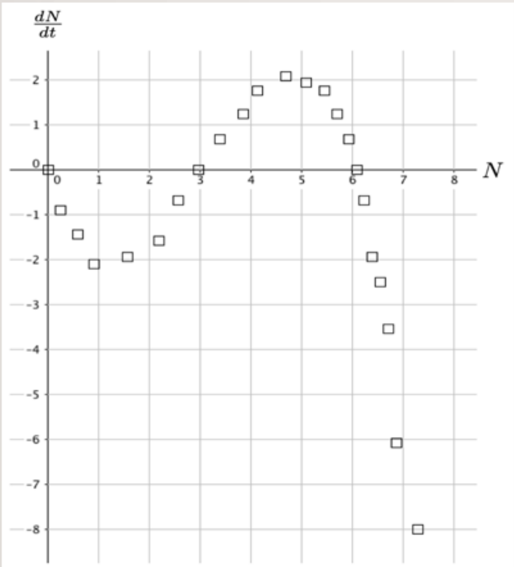
FURTHER USE OF AUTONOMOUS DERIVATIVE GRAPHS

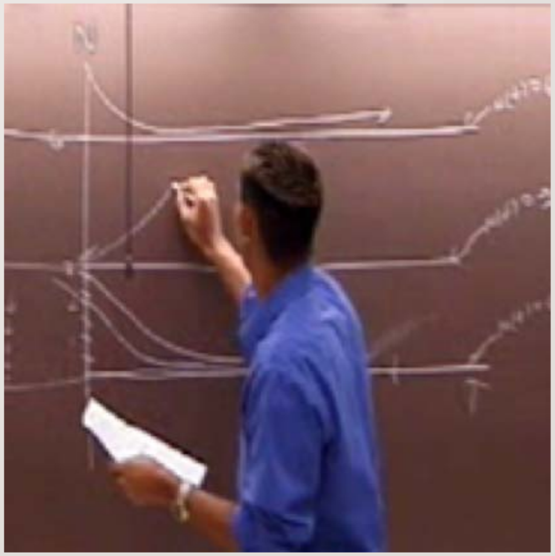
Biologists collected data on the number of particular bug population (N) in a rain forest and created an autonomous derivative graph (dN/dt vs N) of their data so that they could graphically predict the population for a various initial conditions.



Using the graph of dN/dt vs N, sketch graphs of N vs t for each of the following initial conditions:
a) $N(0) = 2$; b) $N(0) = 3$; c) $N(0) = 4$; d) $N(0) = 4.5$; e) $N(0) = 6$; f) $N(0) = 8$

SAMPLE STUDENT SOLUTION





FISH HARVESTING TASK: $\frac{dP}{dt} = 2p(1 - \frac{p}{25})$

A scientist at a fish hatchery has previously demonstrated that the above rate of change equation is a reasonable model for predicting the number of fish that the hatchery can expect to find in their pond.

Recently, the hatchery was bought out by fish.net and the new owners are planning to allow the public to catch fish at the hatchery (for a fee of course). The new owners need to decide how many fish per year they should allow to be harvested. Prepare a report for the new owners that illustrate the implications that various choices for harvesting will have on future fish populations.

Below are possible modifications to the differential equation to account for the new plan. Do you agree with either of these? If yes, explain why. If no, create your own modification and explain your reasoning

$\frac{dP}{dt} = 2P(1 - \frac{P}{25}) - kP$ $\qquad$ $\frac{dP}{dt} = 2P(1 - \frac{P-k}{25})$ $\qquad$ or create your own

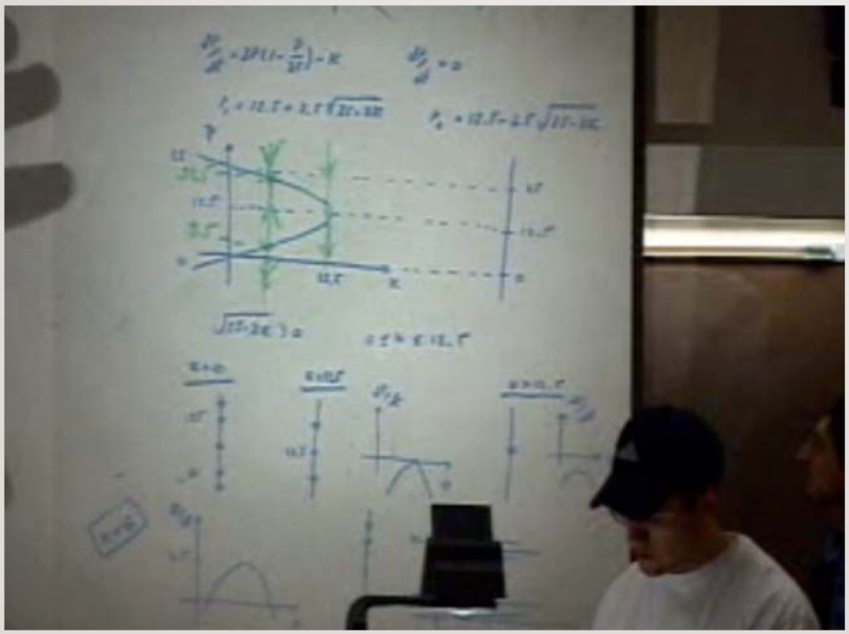
Task presented students with an opportunity to engage in modeling: First have to create a new differential equation to fit the new context

$$\frac{dP}{dt} = 2p(1 - \frac{p}{25}) \longrightarrow \frac{dP}{dt} = 2p(1 - \frac{p}{25}) - k$$

Key features of the task:

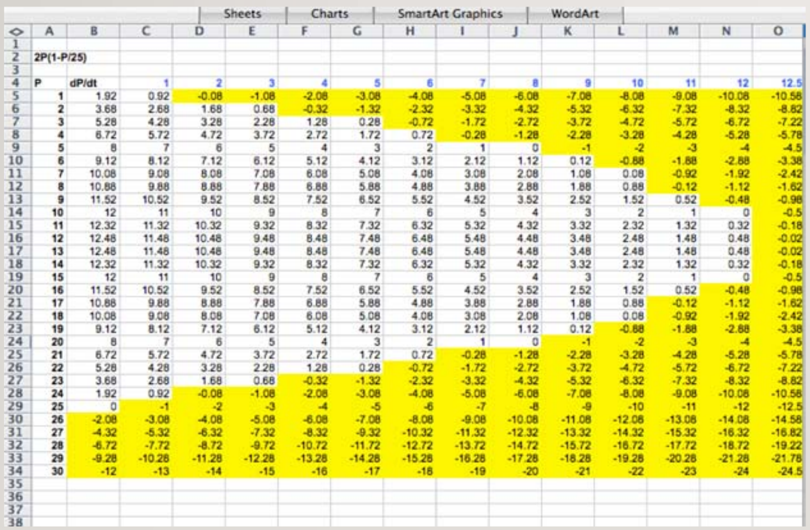
- Did not ask students to find the best harvesting rate
- Limited the report to one page

STUDENT REPORT ELABORATED DURING DISCUSSION



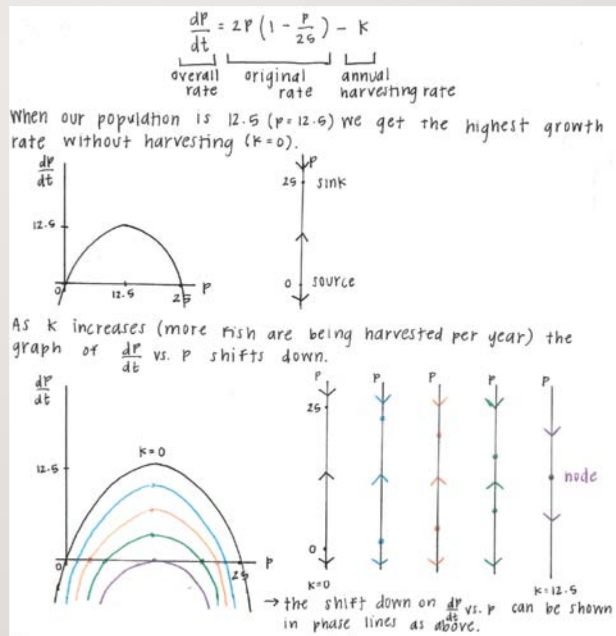
Instructor used student work as the basis for introducing conventional terminology of bifurcation diagram

SAMPLE STUDENT REPORTS



2P(1-P/25)

P	dP/dt	1	2	3	4	5	6	7	8	9	10	11	12	12.5
1	1.92	0.92	-0.08	-1.08	-2.08	-3.08	-4.08	-5.08	-6.08	-7.08	-8.08	-9.08	-10.08	-10.58
2	3.68	2.68	1.68	0.68	-0.32	-1.32	-2.32	-3.32	-4.32	-5.32	-6.32	-7.32	-8.32	-8.82
3	5.28	4.28	3.28	2.28	1.28	0.28	-0.72	-1.72	-2.72	-3.72	-4.72	-5.72	-6.72	-7.22
4	6.72	5.72	4.72	3.72	2.72	1.72	0.72	-0.28	-1.28	-2.28	-3.28	-4.28	-5.28	-5.78
5	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-4.5
6	9.12	8.12	7.12	6.12	5.12	4.12	3.12	2.12	1.12	0.12	-0.88	-1.88	-2.88	-3.38
7	10.08	9.08	8.08	7.08	6.08	5.08	4.08	3.08	2.08	1.08	0.08	-0.92	-1.92	-2.42
8	10.88	9.88	8.88	7.88	6.88	5.88	4.88	3.88	2.88	1.88	0.88	-0.12	-1.12	-1.62
9	11.52	10.52	9.52	8.52	7.52	6.52	5.52	4.52	3.52	2.52	1.52	0.52	-0.48	-0.98
10	12	11	10	9	8	7	6	5	4	3	2	1	0	-0.5
11	12.32	11.32	10.32	9.32	8.32	7.32	6.32	5.32	4.32	3.32	2.32	1.32	0.32	-0.18
12	12.48	11.48	10.48	9.48	8.48	7.48	6.48	5.48	4.48	3.48	2.48	1.48	0.48	-0.02
13	12.48	11.48	10.48	9.48	8.48	7.48	6.48	5.48	4.48	3.48	2.48	1.48	0.48	-0.02
14	12.32	11.32	10.32	9.32	8.32	7.32	6.32	5.32	4.32	3.32	2.32	1.32	0.32	-0.18
15	12	11	10	9	8	7	6	5	4	3	2	1	0	-0.5
16	11.52	10.52	9.52	8.52	7.52	6.52	5.52	4.52	3.52	2.52	1.52	0.52	-0.48	-0.98
17	10.88	9.88	8.88	7.88	6.88	5.88	4.88	3.88	2.88	1.88	0.88	-0.12	-1.12	-1.62
18	10.08	9.08	8.08	7.08	6.08	5.08	4.08	3.08	2.08	1.08	0.08	-0.92	-1.92	-2.42
19	9.12	8.12	7.12	6.12	5.12	4.12	3.12	2.12	1.12	0.12	-0.88	-1.88	-2.88	-3.38
20	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-4.5
21	6.72	5.72	4.72	3.72	2.72	1.72	0.72	-0.28	-1.28	-2.28	-3.28	-4.28	-5.28	-5.78
22	5.28	4.28	3.28	2.28	1.28	0.28	-0.72	-1.72	-2.72	-3.72	-4.72	-5.72	-6.72	-7.22
23	3.68	2.68	1.68	0.68	-0.32	-1.32	-2.32	-3.32	-4.32	-5.32	-6.32	-7.32	-8.32	-8.82
24	1.92	0.92	-0.08	-1.08	-2.08	-3.08	-4.08	-5.08	-6.08	-7.08	-8.08	-9.08	-10.08	-10.58
25	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-12.5
26	-2.08	-3.08	-4.08	-5.08	-6.08	-7.08	-8.08	-9.08	-10.08	-11.08	-12.08	-13.08	-14.08	-14.58
27	-4.32	-5.32	-6.32	-7.32	-8.32	-9.32	-10.32	-11.32	-12.32	-13.32	-14.32	-15.32	-16.32	-16.82
28	-6.72	-7.72	-8.72	-9.72	-10.72	-11.72	-12.72	-13.72	-14.72	-15.72	-16.72	-17.72	-18.72	-19.22
29	-9.28	-10.28	-11.28	-12.28	-13.28	-14.28	-15.28	-16.28	-17.28	-18.28	-19.28	-20.28	-21.28	-21.78
30	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-24.5



REINVENTION OF BIFURCATION DIAGRAM: EMPOWERING LEARNERS THROUGH MODELING

Student reflection:

The fish.net problem was very powerful for me because I was actually able to come up with a diagram that reflected the relationship between any value of the fish population and the parameter…Because I was able to develop a modified bifurcation diagram to answer the fish.net problem, the ideas that followed in class discussion (...) were not additional concepts for me but rather ways of enhancing what I had already done with formal notation.

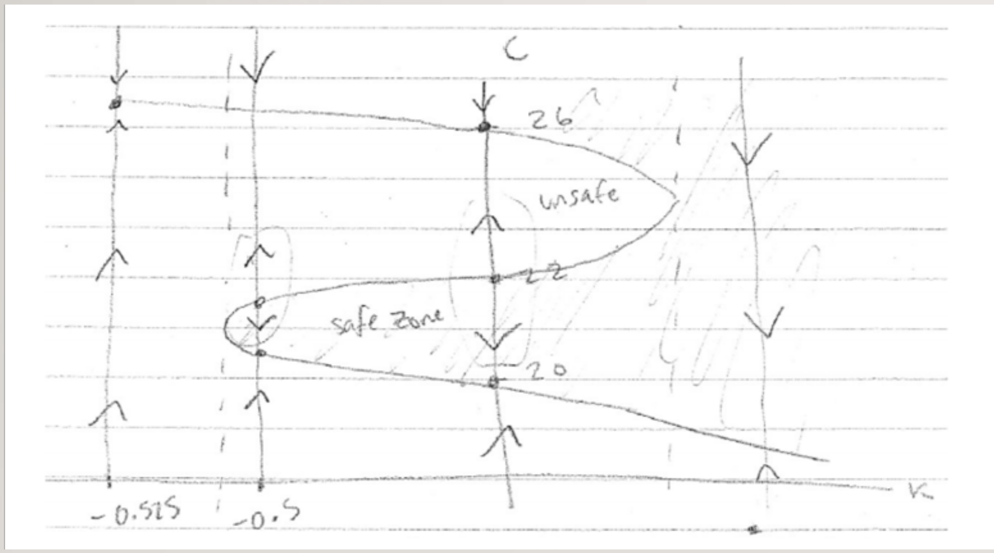
CLIMATE CHANGE PROBLEM

“Future climate change is expected to further disrupt many areas of life, exacerbating existing challenges to prosperity posed by aging and deteriorating infrastructure, stressed ecosystems, and economic inequality. Impacts within and across regions will not be distributed equally. People who are already vulnerable, including lower-income and other marginalized communities, have lower capacity to prepare for and cope with extreme weather and climate-related events and are expected to experience greater impacts.”

Fourth National Climate Assessment
Volume II: Impacts, Risks, and Adaptation in the United States
https://nca2018.globalchange.gov/

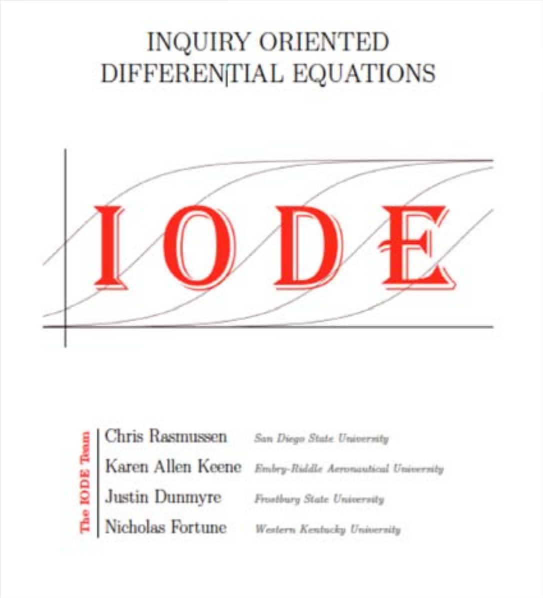
- A group of scientists came up with the following model for this global climate system: $\frac{dC}{dt} = (1/10)(C-20)(22-C)(C-26) - k$, where C is the temperature in Celsius, and k is a parameter that represents governmental regulation of greenhouse gas emissions. Assume the baseline regulation corresponds to $k = 0$, increasing regulation corresponds to increasing k, and the current equatorial temperature is around 20 degrees. To what equatorial temperature will the global climate equilibrate?
- Sketch a bifurcation diagram and use it to describe what happens to the global temperature for various values of k.
- Suppose at the start of a new governmental administration, the temperature at the equator is about 20 degrees Celsius, and $k = 0$. Based on the model and other economic concerns, a government decides to deregulate emissions so that $k = -0.5$. Later, the Smokestack Association successfully lobbied for a 5% change, resulting in $k = -0.525$. Subsequently, a new administration undid that change, reverting to $k = -0.5$, and eventually back to $k = 0$. What is the equilibrium temperature at the equator after all of these changes?
- Use your bifurcation diagram to propose a plan that will return the temperature at the equator to 20 degrees Celsius.

SAMPLE STUDENT WORK



I think this problem really makes you think 'outside the box' not only by getting the answers from numbers, but solving the problem analytically. This problem made me understand more in detail about what is happening with the climate change, if you move the government regulation of greenhouse gas emissions. I like how this helped me understand mathematics in the real world and how using just a graph is so helpful to see the climate change as a result of little or big changes to the greenhouse gas emissions.

THE END – THANKS FOR LISTENING



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