Life After Calculus

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A Note on the Title: “After” rather than “Without”

Calculus as a revolution and era:
- Undeniable historical importance
- Massive reformation of sciences and engineering

However, “calculus”\(^1\) has come to mean...
- A prescribed curriculum genre
- Stewartian calculus: reams of practice problems on arcane topics

The lessons calculus has for us are less relevant now than they once were.

\(^1\): it is worth noting that calculus doesn’t exist everywhere (e.g., Germany).
What does calculus have to offer?

Fundamentally: Calculus is a language for change.

But it is not the only language we could use for change…

“An example was the development of *rere* and *arawhata* as early translations of ‘continuous’ and ‘discrete’ as applied to statistical data, the metaphor being that of a flowing stream or one proceeding in a sequence of waterfalls.” - *(Barton, Fairhall, & Trinick, 1998)*

…nor is change our sole focus.
Overcoming Calculus

Despite its numerous successes, it’s time to overcome calculus—the math that is most relevant to us and our students has shifted.

This doesn’t...

- suggest that differentiation and integration are entirely irrelevant.
- preclude inclusion of aspects of calculus in other contexts.

However, calculus...

1. is less important than we pretend it is.
2. occupies a self-fulfilling role in education.
3. affords and also constrains ways of thinking.
1. Calculus is less relevant than we pretend it is

Most of what we do (in math departments) is in service to other departments (esp. engineering). This stultifies our curriculum.

Example: Finding the centre of mass.

\[ M_x = \int \int_R y \rho(x,y) \, dA, \quad M_y = \int \int_R x \rho(x,y) \, dA \]

\[ M = \int \int_R \rho(x,y) \, dA \]

\[ C_R = \left( \frac{M_y}{M}, \frac{M_x}{M} \right) \]

Method 1: Calculus Apologists/Sadomasochists

Method 2: Sane People
2. Calculus creates its own importance

Q. Is calculus needed for STEM?
   
   A. Yes, because STEM was created with calculus.

Q. Can’t we just teach STEM with calculus as a footnote?
   
   A. Yes.

Example:

$$
\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} = -\eta \delta_i \delta_j
$$
3. Calculus narrows our vision

By taking so much calculus, we begin to think in terms of calculus.

- Calculus is our shared system of metaphors to build off of.
- What might be developed from a different foundation?

Example:

What type of math might the Grand Unified Theory be? (Answer: I don’t know - I took all the calculus courses…)

![Diagram of a Grand Unified Theory (G.U.T.) concept](image)
If not Calculus, then…?

Here’s the answer you’d like me to argue for:

- Discrete mathematics!
- Graph/social network theory!
- Statistics!
- Data!

Here’s the answer I’ll actually give:

Curriculum is a losing game; no matter which curriculum you commit to, you’re wrong.

Rather, our focus as a community ought to be on the core ideas of mathematics.

What are the core ideas that we value, and how are they manifest in disparate contexts?
Example of a Core Idea

Moments:

We can describe an object by how spread out it is from some measure of centrality.
Another Example of a Core Idea

Guess and Check:

“Guess” (i.e., *ansatz*) the answer to a procedural problem, then reverse the procedure on it to check if you get back to where you started.

\[ \int \frac{3}{1-x} \, dx = \]

This is *the* way to antidifferentiate, and does away with 200+ pages of Stewart.
Life After Calculus: a Local(ly unstable) Solution
Math 1 at San José State University

Intended for transition (developmental) students; satisfies general education requirements

Three units:

1. Numbers, Big and Small (Q: How large is a large number?)

2. Uncertainty (Q: Is what you observed what you expected to observe?)

3. Shape (Q: What is the shape of society?)

Q: Why not: 1) Proportional Reasoning, 2) Probability, and 3) Graph Theory?
Unit 1: Numbers

How big is your (future) salary?

$1$ Thousand

$1$ Billion

“About 40 seconds after the explosion, the air blast reached me. I tried to estimate its strength by dropping from about six feet small pieces of paper before, during, and after the passage of the blast wave. Since, at the time, there was no wind I could observe very distinctly and actually measure the displacement of the pieces of paper that were in the process of falling while the blast was passing. The shift was about $2\frac{1}{2}$ meters, which, at the time, I estimated to correspond to the blast that would be produced by ten thousand tons of T.N.T.”

-Enrico Fermi
Heads, Belgium wins - and wins
Charlotte Denny and Sarah Dennis
First published on Thu 3 Jan 2002 21.00 EST

Memo to all teams playing Belgium in the World Cup this year: don’t let them use their own coins for the toss. Mathematicians say the coins issued in the eurozone’s administrative heartland are more likely to land heads up than down.

While the notes which began circulating in the 12 members of the eurozone on January 1 are all the same, the coins show national symbols on one side and a map of Europe on the other. King Albert, who appears on Belgian coins, appears to be a bit of a lightweight, according to Polish mathematicians Tomasz Gliszczynski and Wacław Zawadowski. The two professors and their students at the Podlaska Academy in Siedlce spun a Belgian one euro coin 250 times, and found it landed heads up 140 times. "The euro is struck asymmetrically," Prof Gliszczynski, who teaches statistics, told Germany’s Die Welt newspaper.

The head of the mint said yesterday that the Polish mathematicians’ findings were "just luck". "When the coins were made they were struck in exactly the same way on all sides and the metal was evenly distributed," said Romain Coenen. "I haven’t heard of any problems with the coins."

But a variation of the experiment at the Guardian office suggested that the Polish mathematicians may be right. When tossed 250 times, the one euro coin came up heads 139 times and tails 111. "It looks very suspicious to me," said Barry Blight, a statistics lecturer at the London School of Economics. "If the coin were unbiased the chance of getting a result as extreme as that would be less than 7%."
Unit 3: Shape

The torus
The Friendship Paradox (Feld, 1991)

Q. Ever notice that your friends seem to have more friends than you?
A. That’s because some of them do.

The expected number $E[f]$ of friends of a randomly-chosen person is

$$E[f] = \mu + \frac{\sigma^2}{\mu}$$

The fundamental idea behind the paradox: your social network can be described by how far away social contacts are from the “mean” contact. The paradox is a paradox of moments.
Summary

● Calculus has much to offer
  ○ Historically significant (though other significant developments are not canon)
  ○ Problem solving (though this can be promoted with other, more relevant topics)
  ○ Applicability (sure, but less so as time goes on)

● The centrality of Calculus is in question.
  ○ Does Calculus create its own importance?
  ○ How might we think without calculus?

● What might take its place?
  ○ Let’s not be too concerned about the topics; focus on process and disposition.
  ○ All the good stuff in “calculus” ought to be included throughout various curricula, just don’t call it calculus: that has too much baggage.
A Thanks to the Organizers

Mathematics education is often too “sciency”.

We need less of that and more speculative imagining.

That is: forums like this are badly needed!
Questions?

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