Part 2: Scaffolding in math & stats courses

FYMSiC Online Teaching Meetup

Wednesday, July 27th, 2022 @ 2 p.m. (EDT)

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Scaffolding . . .

Two aspects:

1. the way a course is run, and
2. the delivery of lectures, assessments, course materials, etc.

How does one create balance between too little or too much scaffolding?

Scaffolding . . .

When does scaffolding help or impede learning?

Example:
Read this page/watch video before class; in class, form groups to discuss something or work on activities; then, possibly take a short quiz; then, 10-minute lecturing, then, think-pair-share for three minutes, and then, present your key finding, etc.; after class do ...

From the homework readings . . .

How Scaffolding Instruction Leads to More Student Learning in Math by Kate Gasaway

So what does this mean for classroom teachers? What I’ve read suggests to me that George Pólya’s opening statement from his 1945 book, *How to Solve It*, still holds true:

“One of the most important tasks of the teacher is to help his students. This task is not quite easy; it demands time, practice, devotion, and sound principles. The student should acquire as much experience of independent work as possible, but if he is left alone with his problem without any help, he may make no progress at all. If the teacher helps too much, nothing is left to the student. The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work.”

From the homework readings . . .

Scaffolding Practices that Enhance Math Learning by Julia Anghileri

Characterising Scaffolding

Introducing the metaphor of scaffolding to help explore the nature of adult interactions in children's learning, Wood et al. (1976) identified six key elements:

- *recruitment* – enlisting the learner's interest and adherence to the requirements of the task;
- *reduction in degrees of freedom* – simplifying the task so that feed-back is regulated to a level that could be used for correction;
- *direction maintenance* – (verbal prodder and corrector) keeping the learner in pursuit of a particular objective;
- *marking critical features* – (confirming and checking) accentuating some and interpreting discrepancies;
- *frustration control* – responding to the learner's emotional state;
- *demonstration* – or modelling solution to a task. (p. 98)

A comment about the readings (and others too) . . .

Warning to the readers:

There may be possible biases by the author(s) as negative or no results are often not published !!!

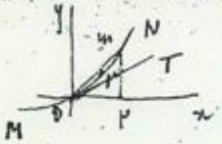
mais pour des fonctions continues quelconques.

Cette limite pourrait avoir pour valeur zéro, aussi que toute autre valeur numérique, positive ou négative d'autres termes, il n'est pas impossible que la courbe coupe précisément l'axe des abscisses au point A, qui point commun d'intersection de cet axe et des deux cou MN, PQ. Nous avons déjà remarqué que, par un cas ég ment exceptionnel, la droite AKL pourrait être une asy tote de la courbe RS, ce qui revient à dire que la limit question aurait pour valeur l'infini positif ou négatif. Il doit résulter aucune restriction dans nos énoncés, car on habitué, en mathémathiques, à regarder zéro et l'in comme des valeurs particulières qui peuvent, aussi bien tout autre, être attribuées à des quantités variables, et conséquent aux limites vers lesquelles ces quantités con gent.

29. Maintenant, envisageons plus spécialement le cas l'on ferait

$$Y = Fx = x,\ u = \frac{fx}{x};$$

et supposons que la courbe $y = fx$ passe par l'origine coordonnées, de manière qu'on ait à la fois $x = o$, $fx =$ Soit NN cette courbe (*fig.* 12), et menons par le point tangente OT : la tangente trigonométrique de l'angle qu droite OT forme avec le demi-axe OX sera précisém la limite dont nous venons de reconnaître l'existence,



M.Cournot, *Traité Elémentaire de la Théorie des Fonctions et du Calcul Infinitésimal*, L. Hachette, 1857

Compared to . . .

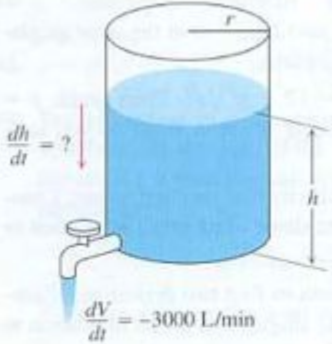


FIGURE 3.45 The rate of change of fluid volume in a cylindrical tank is related to the rate of change of fluid level in the tank (Example 1).

EXAMPLE 1 How rapidly will the fluid level inside a vertical cylindrical tan[k] … we pump the fluid out at the rate of 3000 L/min?

Solution We draw a picture of a partially filled vertical cylindrical tank, callin[g] … dius r and the height of the fluid h (Figure 3.45). Call the volume of the fluid V.

As time passes, the radius remains constant, but V and h change. We think of … as differentiable functions of time and use t to represent time. We are told that

$$\frac{dV}{dt} = -3000.$$

We pump out at the rate of 3000 L/min. The rate is nega[tive] because the volume is decrea[sing].

We are asked to find

$$\frac{dh}{dt}.$$

How fast will the fluid level d[rop]…

All decisions have already been made (labels, variables, picture drawn) – is this the best way to present an example?

[Hass, Weir, Thomas, *University Calculus*, Pearson, 2007]

Proof According to the definition of the derivative, we have

$$\frac{d}{dx}(\ln x) = \lim_{\Delta x \to 0} \frac{\ln(x + \Delta x) - \ln x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \ln\left(\frac{x + \Delta x}{x}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \ln\left(1 + \frac{\Delta x}{x}\right)$$

Let $h = \frac{x}{\Delta x}$, so $\Delta x = \frac{x}{h}$.
As $\Delta x \to 0$, $h \to +\infty$.

$$= \lim_{h \to +\infty} \frac{h}{x} \ln\left(1 + \frac{1}{h}\right)$$

$$= \frac{1}{x}\left[\lim_{h \to +\infty} h \ln\left(1 + \frac{1}{h}\right)\right]$$

$$= \frac{1}{x}\left[\lim_{h \to +\infty} \ln\left(1 + \frac{1}{h}\right)^h\right]$$

Power rule for logarithms

$$= \frac{1}{x}\left[\ln \lim_{h \to +\infty} \left(1 + \frac{1}{h}\right)^h\right]$$

Since $\ln x$ is continuous, we use the composition limit rule.

$$= \frac{1}{x} \ln e$$

Definition of e

$$= \frac{1}{x}$$

$\ln e = 1$

[Bradley, Smith, *Calculus, Second Edition*, Prentice Hall, 1999]

71. **Sue's Calculus Book.** Sue throws her calculus book out her dormitory window straight up with a velocity of 48 feet per second. (See Figure 29.) Assuming that her dormitory room is 40 feet above the ground, the height of her book after t seconds will be (neglecting air friction)

$$s(t) = -16t^2 + 48t + 40$$

[Farlow, Haggard, *Calculus and Its Applications, Second Edition*, McGraw Hill, 1990]

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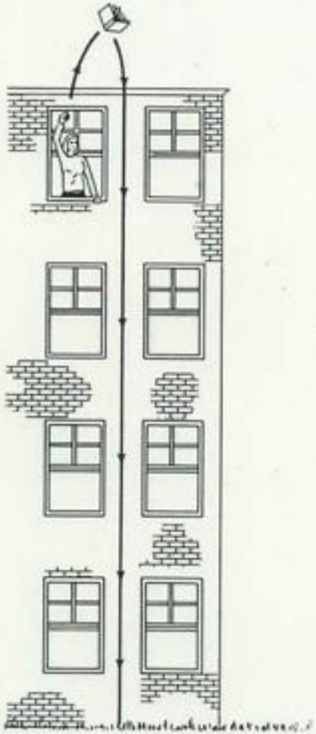

Figure 29
Typical falling object

Discussion questions . . .

1. Relation between scaffolding and developing one's agency ("just tell me what to do" vs developing independence). How much scaffolding is too much scaffolding? In other words, how does scaffolding affect students' agency and maturity (in math, and in general)?

2. Scaffolding in level 1 courses vs upper years - the same or different?

3. Does scaffolding help or hinder the development of problem-solving skills?

4. How to effectively scaffold "theoretical" content in math, such as using or interpreting a theorem or proof using epsilon-delta?