Coq Reference Sheet

Manipulating Goals
intros. If your goal is \( \forall n, P \rightarrow Q \), introduce a variable \( n \) and a hypothesis \( P \), and change your goal to \( Q \). Usually you can start every proof with intros.

split.
– Your goal is \( P \land Q \) and you want to first prove \( P \), then prove \( Q \).
– Your goal is \( P \leftrightarrow Q \) and you want to first prove \( P \rightarrow Q \), then \( Q \rightarrow P \).

left. Your goal is \( P \lor Q \), and you want to prove \( P \).

right. Your goal is \( P \lor Q \), and you want to prove \( Q \).

exists \( x \). Your goal is \( \exists n, P \), and you want to prove that the choice \( n = x \) satisfies \( P \).

assert \( P \). Replace the current goal with \( P \). After proving \( P \), it appears as a new hypothesis, and you must then prove your original goal.

contradiction \( H \). If \( H \) is of the form \( \neg P \), your goal will change to \( P \).

Manipulating Hypotheses
apply \( H \). Given \( H : P \rightarrow Q \), if your goal is \( Q \), change your goal to \( P \).
apply \( (H \times y) \). You have \( H : \forall m n; P \rightarrow Q \), and you want to use \( H \) with \( m = x \) and \( n = y \).
apply \( H1 \text{ in} H2 \). Given \( H1 : P \rightarrow Q \) and \( H2 : P \), change \( H2 \) to \( Q \).

destruct \( H \). (Always replaces \( H \).)
– Given \( H : P \land Q \), replace \( H \) with two new hypotheses \( P \) and \( Q \).
– Given \( H : P \lor Q \), split up your proof into two cases. In Case 1, you are given \( H : P \) (and you must prove your theorem), and in Case 2, you are given \( H : Q \) (and you must prove your theorem).
– Given \( H : \exists n, P \), create a new variable \( x \) and replace \( H \) with a new hypothesis stating that \( P \) holds for \( x \).

Creating New Hypotheses
pose proof (classic \( P \)). If you need to do case analysis on an arbitrary statement \( P \), the command introduces a new hypothesis \( H : P \lor \neg P \). Usually this command is followed by the application of destruct \( H \).

Other Commands
unfold not. changes \( \neg P \) to \( P \rightarrow \text{False} \).

exact. Tell Coq that one of the hypotheses exactly matches the goal. This proves the goal.

contradiction. Tell Coq that there are two logically contradictory hypotheses. This shows that the current case being considered in the proof can never arise.

Warning: A hypothesis which contradicts a goal is NOT A CONTRADICTION! Contradiction means that two hypotheses contradict each other.