## Coq Reference Sheet

**Manipulating Goals**

**intros.** If your goal is ∀ n, P → Q, introduce a variable n and a hypothesis P, and change your goal to Q. Usually you can start every proof with intros.

**split.**

– Your goal is P ⍰ Q and you want to first prove P, then prove Q.

– Your goal is P ↔ Q and you want to first prove P → Q, then Q → P.

**left.** Your goal is P ∨ Q, and you want to prove P.

**right.** Your goal is P ∨ Q, and you want to prove Q.

**exists** x**.** Your goal is ∃ n, P, and you want to prove that the choice n = x satisfies P.

**assert P.** Replace the current goal with P. After proving P, it appears as a new hypothesis, and you must then prove your original goal.

**contradiction** H**.** If H is of the form ¬P, your goal will change to P.

**Manipulating Hypotheses**

**apply** H**.** Given H : P → Q, if your goal is Q, change your goal to P.

**apply (**H x y**).** You have H : ∀ m n; P → Q, and you want to use H with m = x and n = y.

**apply** H1 **in** H2**.** Given H1 : P → Q and H2 : P, change H2 to Q.

**destruct H.** (Always replaces H.)

– Given H : P ⍰ Q, replace H with two new hypotheses P and Q.

– Given H : P ∨ Q, split up your proof into two cases. In Case 1, you are given H : P (and you must prove your theorem), and in Case 2, you are given H : Q (and you must prove your theorem).

– Given H : ∃ n, P, create a new variable x and replace H with a new hypothesis stating that P holds for x.

**Creating New Hypotheses**

**pose proof (**classic P**).** If you need to do case analysis on an arbitrary statement P, the command introduces a new hypothesis H : P ∨ ¬P. Usually this command is followed by the application of destruct H.

**Other Commands**

**unfold not.** changes ¬P to P → False.

**exact.** Tell Coq that one of the hypotheses exactly matches the goal. This proves the goal.

**contradiction.** Tell Coq that there are two logically contradictory hypotheses. This shows that the current case being considered in the proof can never arise.

**Warning:** A hypothesis which contradicts a goal is NOT A CONTRADICTION! Contradiction means that two hypotheses contradict each other.