Student (quietly): I am somewhat confused with today’s lecture about differentials. I am not sure if I understand what’s going on there.

Instructor (speaking, writing, and drawing authoritatively): It’s simple... You have a differentiable function \( y = f(x) \), you fix a number \( x \) in the domain of \( f \), and find the linearization of \( f \) at \( x \): \( L(t) = f(x) + f'(x)(t - x) \). Next you look at the change of \( L \): \( L(t) - L(x) = L(t) - f(x) = f'(x)(t - x) \). You see this on the graph?

Student (impatiently): Yes, I got that part. This is what confuses me. You decided to write \( dx = t - x \) and \( dy = L(t) - L(x) \). Then, you wrote \( dy = f'(x)dx \) and called this the differential.

Instructor (confidently): Yes, that’s what I said.

Student (softly): I really don’t understand with how many independent variables we are dealing with in the expression \( dy = f'(x)dx \). I think that “\( x \)” has to be a variable, but “\( t \)” looks to me like a variable too. I guess that \( x \) and \( t \) do not depend on each other? It looks like that they play different roles in the definition of the differential. And the variable \( t \) is somehow hidden in \( dx \). It didn’t disappear, did it? I feel like \( dx \) would change if I change the value of \( x \), but it will also change if I change the value of \( t \). Is this right? We have never mentioned a function of two variables in our Calculus class.

Instructor (thoughtfully): Yes, you will study functions of two or more variables in Calculus 3. Well, honestly, I don’t remember thinking about differentials in this way. What you said sounds reasonable, but let me think about that a bit more.

Student (excitedly): Thank you, Professor! But there is one more thing that bugs me. Is this “\( y \)” in \( dy \) the same “\( y \)” as in \( y = f(x) \)? I am asking this because “\( dy \)” depends on \( x \) and \( t \), while “\( y \)” depends on \( x \) only.
Bibliography


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