A Calculus Vignette

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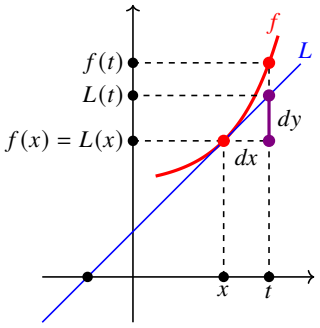
Student (quietly): I am somewhat confused with today's lecture about differentials. I am not sure if I understand what's going on there.

Instructor (speaking, writing, and drawing authoritatively): It's simple... You have a differentiable function $y = f(x)$, you fix a number x in the domain of f, and find the linearization of f at x: $L(t) = f(x) + f'(x)(t - x)$. Next you look at the change of L: $L(t) - L(x) = L(t) - f(x) = f'(x)(t - x)$. You see this on the graph?



Student (impatiently): Yes, I got that part. This is what confuses me. You decided to write $dx = t - x$ and $dy = L(t) - L(x)$. Then, you wrote $dy = f'(x)dx$ and called this the *differential*.

Instructor (confidently): Yes, that's what I said.

Student (softly): I really don't understand with how many independent variables we are dealing with in the expression $dy = f'(x)dx$. I think that "x" has to be a variable, but "t" looks to me like a variable too. I guess that x and t do not depend on each other? It looks like that they play different roles in the definition of the differential. And the variable t is somehow hidden in dx. It didn't disappear, did it? I feel like dx would change if I change the value of x, but it will also change if I change the value of t. Is this right? We have never mentioned a function of two variables in our Calculus class.

Instructor (thoughtfully): Yes, you will study functions of two or more variables in Calculus 3. Well, honestly, I don't remember thinking about differentials in this way. What you said sounds reasonable, but let me think about that a bit more.

Student (excitingly): Thank you, Professor! But there is one more thing that bugs me. Is this "y" in dy the same "y" as in $y = f(x)$? I am asking this because "dy" depends on x and t, while "y" depends on x only.

Bibliography

Apostol, T. M. (1957). Mathematical Analysis - A Modern Approach to Advanced Calculus. *Addison-Wesley Publishing Company*, Massachusetts.