APPLYING COGNITIVE SCIENCE PRINCIPLES FOR EFFECTIVE MATH TEACHING

Anna Stokke
University of Winnipeg

Podcast: Chalk & Talk with Anna Stokke
chalkandtalkpodcast.podbean.com

e-mail: a.stokke@uwinnipeg.ca
YouTube: @annas9312

A FEW NOTES

- Presentation applies to: novice learners, average students
- Disclaimer: not a cognitive scientist
- Podcast episodes with: Dan Willingham, Paul Kirschner, Barb Oakley, Greg Ashman, Zach Groshell, Patrice Bain
- This presentation has tests 😀

TRY TO REMEMBER

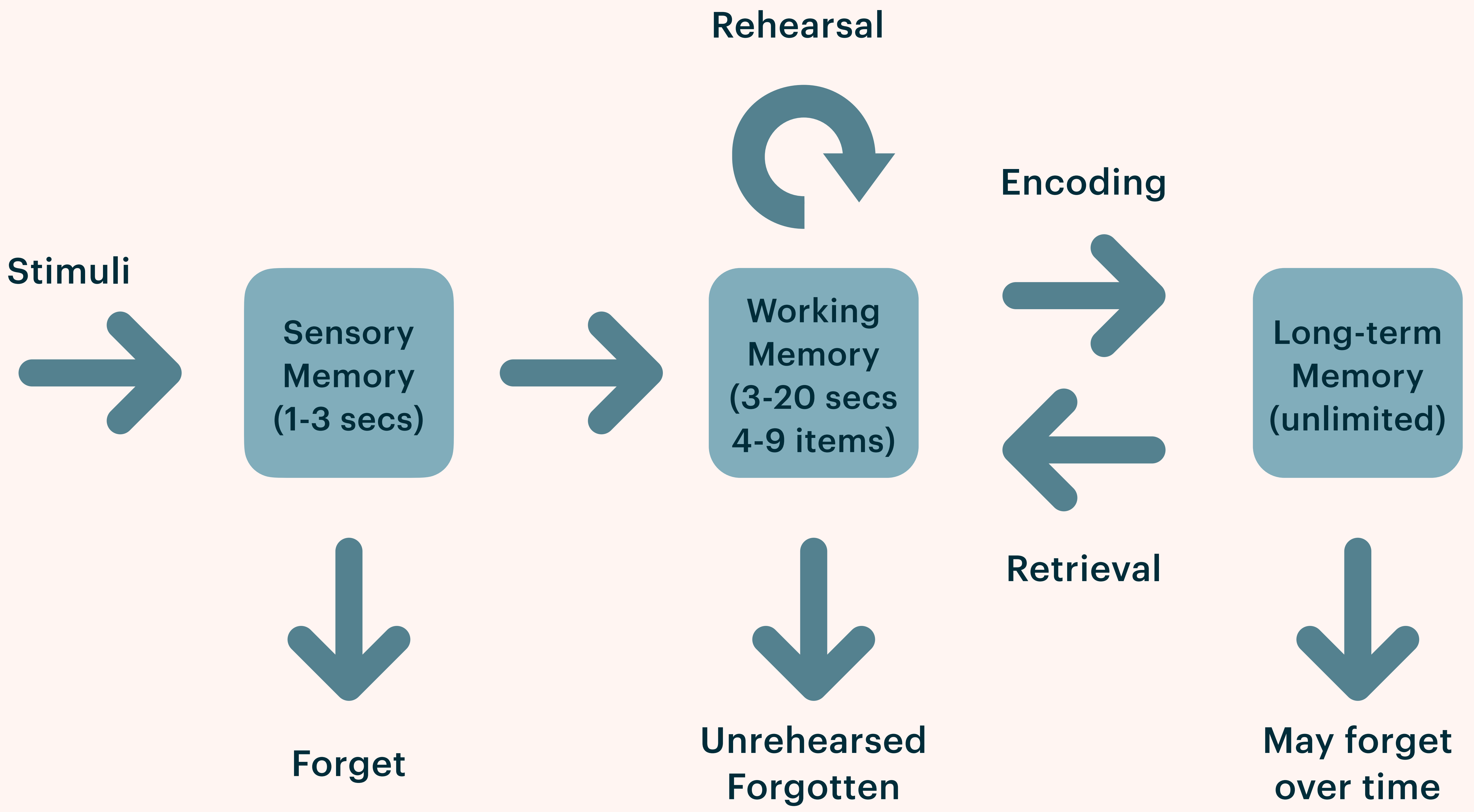
TKM UIK IOB AEI ADB

HOW MANY CAN YOU REMEMBER?

TRY TO REMEMBER

TMI BCE IOU AKA IDK

HOW MANY CAN YOU REMEMBER?



Adapted from Atkinson, R.C. and Shiffrin, R.M. (1968). "Human memory: A Proposed System and its Control Processes". In Spence, K.W. and Spence, J.T. *The psychology of learning and motivation,* (Volume 2). New York: Academic Press. pp. 89–195.

DEFINITION: LEARNING IS A CHANGE IN LONG-TERM MEMORY

- KIRSCHNER, SWELLER, CLARK

COGNITIVE LOAD THEORY (CLT)

BASIC IDEA

- Align teaching with cognitive architecture
- Working memory: limited amount of new info
- Long-term memory: no known maximum capacity
- Automated schema: take less space in WM (e.g. memorized derivatives, techniques you know)

Working memory bottleneck

COGNITIVE LOAD

- Cognitive load: working memory required for learning task
- Cognitive overload : cognitive load exceeds working memory capacity

CHECKING IN

What is Kirschner, Sweller, Clark's definition of learning?

a) engaging in a learning opportunity

b) a change in long-term memory

c) being taught how to do something

d) discovering an unknown concept

EXTRANEOUS LOAD

- Bad
- Working memory used by things unrelated to learning
- Aim: reduce extraneous load

E.g. cell phones, noise

STRATEGIES FOR INSTRUCTION

- **REDUCE EXTRANEOUS LOAD**
- **DON’T OVERWHELM WORKING MEMORY**
- **USE STRATEGIES THAT CAUSE CHANGES IN LONG-TERM MEMORY**

OUTLINE

1. Scaffolding
2. Worked example effect
3. Retrieval practice
4. Spaced practice
5. Interleaving

SCAFFOLDING

SCAFFOLDING

- Increase difficulty and variability gradually
- Why?   Working memory!
- How much? Depends on previous knowledge of learner, difficulty of topic
- Beware: expert-induced blindness / curse of knowledge

CHAIN RULE SCAFFOLDING

Chain rule (tough)

Find the derivative: $y = \sqrt{x^2 + x\sin(e^{7x} + 5x^2)}$

Chain rule (scaffolding)

Find the derivative:

$$y = e^{7x} \to y = \sin(5x^2) \to y = \sqrt{x^2 + 3x} \to y = \sqrt{\sin(e^x)} \to \cdots$$

IVT SCAFFOLDING

IVT (tough)

Show that the equation $4\sin x + 4\cos^2 x = x^3$ has at least two solutions.

IVT (better to start here)

Show that the equation $x^3 + x - 9 = 0$ has at least one solution on $(1, 2)$.

CHECKING IN

What is *extraneous load*?

a) loading up on extra practice problems

b) pulling things out of long-term memory

c) working memory used by things unrelated to learning

d) an excess amount of work

WORKED EXAMPLE EFFECT

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- **Early stage: worked examples from expert**
- **Intermediate: students study examples, use self-explanations**
- **Final stages: independent problem solving (goal)**
- **Best: alternate worked examples (instructor) and problem solving (student)**

Worked example approach (to start, novice learner)

Here is my correct solution to this problem.

Show that the equation $x^3 + x - 9 = 0$ has at least one solution on $(1, 2)$.

Let $f(x) = x^3 + x - 9$.

Since f is a polynomial, it is continuous on $[1,2]$.

$f(1) = 1^3 + 1 - 9 = -7 < 0$

$f(2) = 2^3 + 2 - 9 = 1 > 0$

By the IVT, there is some $c \in (1,2)$ such that $f(c) = 0$.

Therefore the equation $x^3 + x - 9 = 0$ has at least one solution on $(1,2)$.

Your turn!

Show that the equation $2x^3 - 5x + 1 = 0$ has at least one solution on $(0, 1)$.

Worked example fading (to start, novice learner)

Here is my correct solution to this problem.

Show that the equation $x^3 + x - 9 = 0$ has at least one solution on $(1, 2)$.

Let $f(x) = x^3 + x - 9$.

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$f(2) = 2^3 + 2 - 9 = 1 > 0$

By the IVT, there is some $c \in (1,2)$ such that $f(c) = 0$.

Therefore the equation $x^3 + x - 9 = 0$ has at least one solution on $(1,2)$.

Finish the problem.

Show that the equation $2x^3 - 5x + 1 = 0$ has at least one solution on $(0, 1)$.

Let $f(x) = 2x^3 - 5x + 1$.

f is a polynomial so f is continuous on $[0,1]$

$f(0) = 1 > 0$

$f(1) = 2(1) - 5(1) + 1 = -2 < 0$

Finish the problem.

Show that the equation $x^4 + x - 3 = 0$ has at least one solution on $(1, 2)$.

Let $f(x) = x^4 + x - 3$.

Since f is a polynomial, it is continuous on $[1,2]$.

Your turn!

Show that the equation $x^3 + 3x - 2 = 0$ has at least one solution on $(0, 1)$.

NOTES

- Previous is an example of where to start with a novice learner
- Novice learners need a lot of guidance; WM can get overwhelmed quickly with lots of steps
- Increase difficulty, variability
- Aim: independent problem solving

WHY DO STUDENTS HAVE TROUBLE WHEN WE VARY PROBLEMS?

“Every field has this problem…What you’re asking them to do is really difficult. [Experts have no problem] because we’ve done a lot of [those types of problems].

Is there anything you can do that's a little faster? The best answer we have is problem comparison. So, if you give students several different versions of a problem where they need to calculate [something] and ask the students not just to work the problems, but to compare the problems, this is a variation. You’re finding the elements that the problems have in common, but have the same underlying mathematics. You’re highlighting for them and getting them to think about the relevant mathematics.”

-Daniel Willingham

CHECKING IN

What is *scaffolding*?

a) providing worked examples with practice

b) modelling examples with pictures

c) moving information from working memory to long-term memory

d) gradually increasing difficulty and variability of problems

RETRIEVAL PRACTICE

RETRIEVAL: THE PROCESS OF PULLING SOMETHING OUT OF MEMORY

"A GREAT DEAL HAS BEEN WRITTEN ABOUT THE IMPACT OF RETRIEVAL PRACTICE ON MEMORY. THAT'S BECAUSE THE EFFECT IS SIZEABLE, IT HAS BEEN REPLICATED MANY TIMES AND IT SEEMS TO LEAD NOT JUST TO BETTER MEMORY BUT DEEPER MEMORY THAT SUPPORTS TRANSFER."

-DANIEL WILLINGHAM

RETRIEVAL PRACTICE IN MATH

- Recalling
  - a fact or definition
  - a problem technique
  - a proof technique
- Spend time getting info INTO memory AND time pulling it out
- Beware: expert-induced blindness; novices forget quickly

GOOD EXPLICIT INSTRUCTION IS INTERACTIVE AND PROVIDES A LOT OF OPPORTUNITIES FOR RETRIEVAL

RETRIEVAL PRACTICE IDEAS

- Instead of “recall we learned yesterday that…” ask students to recall
- Ask lots of questions
- iClickers or index cards
- Daily retrieval practice at beginning of class
- Frequent low-stakes assessments (e.g. quizzes, assignments)
  - help students learn
  - reduce stress for high-stakes assessments

Visit: retrievalpractice.org

OTHER RETRIEVAL PRACTICE STRATEGIES

- Tests/exams
- Students: you need to be able to work the problems yourself
- Answers to practice problems vs. full solutions

Aside: Worried about stress/anxiety?

- Ep. 14 with Dan Rosen
- Ep. 17 with Robin Codding

CHECKING IN

Which of the following is an example of something that might cause *cognitive overload*?

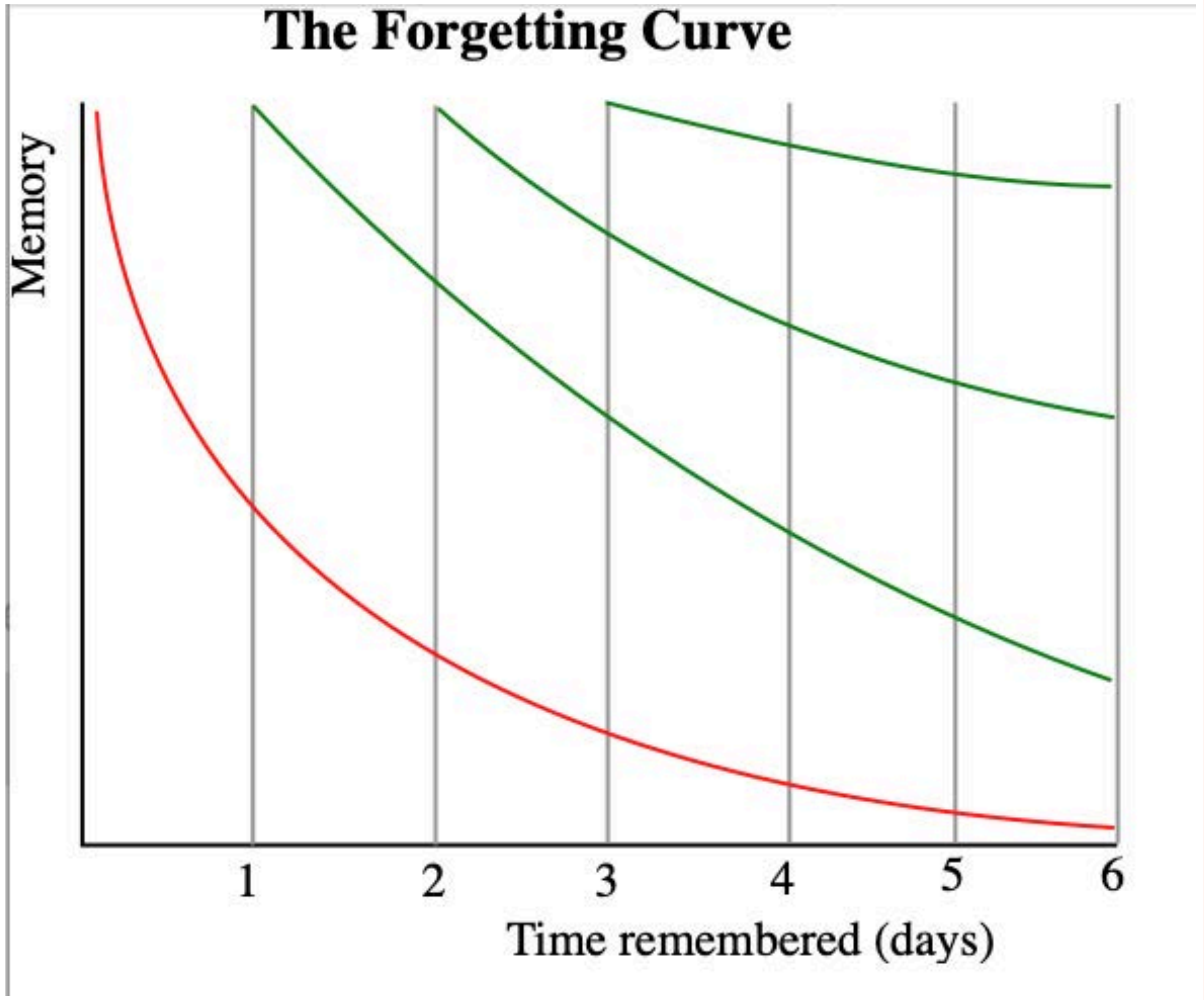
a) a new concept introduced with too many steps

b) too few practice problems

c) no pictures on the page

d) prof speaking too slowly

SPACED PRACTICE



The Forgetting Curve
Memory
1
2
3
4
5
6
Time remembered (days)

SPACED PRACTICE

- Spaced practice: opposite of cramming
- Time delay between practice sessions for new skills
- Forces slight forgetting, relearning, strengthening long-term memory
- Goal: long-term retention

SPACED PRACTICE INSTRUCTIONAL STRATEGIES

- **Cumulative tests**
- **Incorporate topics covered earlier into practice/assignments**
- **Schedule spaced practice before exams**
- **Tell students about it!**

INTERLEAVING

INTERLEAVING

Blocked practice: practice same type of question

Interleaving: mix up similar problems

Hybrid approach is best (blocking → interleaving)

- Interleaving: Problems look similar, students choose and use a strategy
- Spacing: problems don't have to be similar

MATH-1103(3) Problems – Limits

Mini-videos on limits

Factoring limit 1
Limit 2
Long division limit 3
Rationalization limit 4
Absolute value limit 5

Make sure:

- ☐ You use equals signs.
- ☐ You place limit symbols in the right places (write your solutions in the same way your professor does - follow your class notes).
- ☐ Consult your class notes and the videos.
- ☐ Star problems that you find tricky; come back to them later and make sure you can do them on your own.

Evaluate the following limits.

A)

1. $\lim_{x\to 2}(x^3-2x+1)$
2. $\lim_{x\to 0}\dfrac{x^2-2x-3}{x^2+4x+3}$
3. $\lim_{x\to 1}\dfrac{x-1}{x^2+x-2}$
4. $\lim_{x\to 2}\dfrac{x^2-4}{x^2+x-2}$
5. $\lim_{x\to 1}\dfrac{x^2-1}{x^2+x-2}$
6. $\lim_{x\to 1}\dfrac{x^2-2x+1}{x^2+3x+2}$
7. $\lim_{x\to 4}\dfrac{x^2-16}{x^2-3x-4}$
8. $\lim_{x\to -2}\dfrac{x^3+3x^2-4}{x^2-4}$
9. $\lim_{x\to -2}\dfrac{3x^2+5x-2}{4x^3+12x^2-7x-30}$
10. $\lim_{x\to 3}\dfrac{2x^3-6x^2+x-3}{x^2-9}$

A. Blocked

B)

1. $\lim_{x\to 1}\left(x^2-x+\sqrt{2x-1}\right)$
2. $\lim_{x\to 3}\sqrt{2x+3}$
3. $\lim_{x\to 0}\dfrac{\sqrt{9+x}-3}{x}$
4. $\lim_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt{1-3x}}{x}$
5. $\lim_{x\to 1}\dfrac{\sqrt{x^2+9}-3}{x}$
6. $\lim_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x-2}$
7. $\lim_{x\to -3}\dfrac{x+\sqrt{6-x}}{x+3}$
8. $\lim_{x\to 2}\dfrac{3-\sqrt{2x^2+1}}{x^2-4}$
9. $\lim_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

B. Blocked

C)

1. $\displaystyle\lim_{x\to 2^+} \frac{|x-2|}{2-x}$
2. $\displaystyle\lim_{x\to 2} \frac{|x-2|}{2-x}$
3. $\displaystyle\lim_{x\to -2^+} \frac{|x+2|}{x^2-4}$
4. $\displaystyle\lim_{x\to 1} \frac{|x-1|}{x}$
5. $\displaystyle\lim_{x\to 0^+} \frac{x^2-3x}{|x|}$
6. $\displaystyle\lim_{x\to 2^+} \frac{x^2+x-6}{|x-2|}$
7. $\displaystyle\lim_{x\to -2} \frac{|x-1|}{x}$
8. $\displaystyle\lim_{x\to 0^-} \frac{x^2-3x}{|x|}$
9. $\displaystyle\lim_{x\to 2^-} \frac{x^2+x-6}{|x-2|}$
10. $\displaystyle\lim_{x\to 2} \frac{2-|x|}{2+x}$
11. $\displaystyle\lim_{x\to -5^+} \frac{|x+5|}{x^2-3x-40}$
12. $\displaystyle\lim_{x\to 3^-} \frac{|3-x|}{x^3-27}$
13. $\displaystyle\lim_{x\to -2^-} \frac{4-x^2}{|x+2|}$
14. $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{|x|}-\frac{1}{x}\right)$
15. $\displaystyle\lim_{x\to -3^+} \frac{5x+15}{|x+3|}$
16. $\displaystyle\lim_{x\to 2} \frac{|x+4|}{|3-x|}$
17. $\displaystyle\lim_{x\to 4} \frac{2x-8}{|4-x|}$

C. Blocked

D)

1. $\displaystyle\lim_{x\to 2} \left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$
2. $\displaystyle\lim_{x\to 0} \left(\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x}\right)$
3. $\displaystyle\lim_{x\to 0} \frac{(6+x)^2-36}{x}$
4. $\displaystyle\lim_{x\to -3} \left(\frac{6}{x^2-9}+\frac{1}{x+3}\right)$
5. $\displaystyle\lim_{x\to 5} \left(\frac{\frac{1}{x}-\frac{1}{5}}{x-5}\right)$
6. $\displaystyle\lim_{x\to 2} \frac{1}{x-2}\left(\frac{1}{x+2}-\frac{1}{4}\right)$

D. Blocked

E)

1. $\displaystyle\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$
2. $\displaystyle\lim_{x\to 3} \frac{x^2-9}{x^2+x-12}$
3. $\displaystyle\lim_{x\to 0} \frac{|x-2|}{2-x}$
4. $\displaystyle\lim_{x\to 1} \frac{\ln(x^2)}{6}$
5. $\displaystyle\lim_{x\to 1} \frac{x-1}{x^3-4x^2+4x-1}$
6. $\displaystyle\lim_{x\to 2} \left(\frac{3}{x^2-x-2}-\frac{1}{x-2}\right)$
7. $\displaystyle\lim_{x\to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
8. $\displaystyle\lim_{x\to -2} \frac{x^2+2x}{x^2-4}$
9. $\displaystyle\lim_{x\to 1} \frac{(x^2-1)^2}{x^3-2x^2+x}$
10. $\displaystyle\lim_{x\to -2} \frac{x^4-16}{x^3+8}$
11. $\displaystyle\lim_{x\to 3} \frac{1}{x-3}\left(\frac{x}{x-1}-\frac{3}{2}\right)$
12. $\displaystyle\lim_{x\to 3} \frac{9-x^2}{x^3-4x^2+2x+3}$
13. $\displaystyle\lim_{x\to 3^-} \frac{x^2-x-6}{|x-3|}$
14. $\displaystyle\lim_{x\to 2\pi} \frac{\cos(x-\pi)}{x}$
15. $\displaystyle\lim_{x\to 0} \frac{x^2+3x}{(x+2)^2-(x-2)^2}$
16. $\displaystyle\lim_{x\to 0} \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{x^2}$
17. $\displaystyle\lim_{x\to 3} \frac{x^3-27}{9-x^2}$
18. $\displaystyle\lim_{x\to \ln 2} \frac{e^x}{3}$

E. Interleaved

CHECKING IN

What is *interleaving*?

a) timed delay in practicing new skills

b) pulling something out of long-term memory

c) sequencing problems from simple to difficult

d) mixing up similar problems so that students have to choose a strategy

SUMMARY

SUMMARY

- Not exhaustive
- Some strategies that consider cognitive architecture
  - scaffolding
  - worked example effect
  - retrieval practice
  - interleaving
  - spaced practice

RECOMMENDED READING / LISTENING

- Outsmart Your Brain: Why Learning is Hard and How you Can Make it Easy, Daniel Willingham
- Powerful Teaching: Unleash the Science of Learning, Pooja Agarwal & Patrice Bain
- How Teaching Happens, Seminal Works in Teaching and Teacher Effectiveness and What They Mean in Practice, Kirschner, Henrick, Heal
- How Learning Happens, Seminal Works in Educational Psychology and What They Mean in Practice, Kirschner & Hendrick
- A Mind for Numbers: How to Excel at Math and Science, Barbara Oakley
- Uncommon Sense Teaching, Barbara Oakley
- A Little Guide for Teachers: Cognitive Load Theory, Greg Ashman
- Good summary: Cognitive load theory: research that teachers really need to understand, Centre for Education Statistics and Evaluation, NSW Department of Education (2017)

RECOMMENDED EPISODES FOR POST-SECONDARY

chalkandtalkpodcast.podbean.com (or wherever you get your podcasts)

- **Ep 2. Evidence-based teaching strategies with Paul Kirschner**
- **Ep 7. How to excel in math and other tough subjects with Barb Oakley**
- **Ep 11. California's math controversy with Jelani Nelson**
- **Ep 14. Stress and learning with Dan Rosen**
- **Ep 15. Modern relevance in the math curriculum with Brian Cornad**
- **Ep 16. Applying cognitive science to education with Daniel Willingham**
- **Ep 17. Do timed tests cause math anxiety? with Robin Codding**

THANK YOU!